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# NAVAL RESEARCH LOGISTICS QUARTERLY

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# NAVAL RESEARCH LOGISTICS QUARTERLY


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The Naval Research Logistics Quarterly is devoted to the dissemination of scientific information in logistics and will publish research and expository papers, including those in certain areas of mathematics, statistics, and economics, relevant to the over-all effort to improve the efficiency and effectiveness of logistics operations.

Manuscripts submitted for publication should be typewritten, double-spaced, and the author should retain a copy. Manuscripts and other items for publication should be sent to the Managing Editor at the above address. Authors are entitled to receive 25 free reprints of their published papers and may obtain additional copies at cost.

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The Naval Research Logistics Quarterly is published by the Office of Naval Research in the months of March, June, September, and December and is distributed to individuals and organizations interested in logistics.


The views and opinions expressed in the articles in this Quarterly are those of the authors and not necessarily those of the Office of Naval Research.

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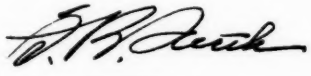
## A MESSAGE FROM REAR ADMIRAL F. R. FURTH

Chief of Naval Research

With this issue the Office of Naval Research launches the NAVAL RESEARCH LOGISTICS QUARTERLY. The decision to initiate this journal has resulted from the need to provide an outlet for the papers on logistics which now appear in widely scattered journals or inaccessible reports. Due to the lack of an appropriate journal in this rapidly growing field, important work is missed by individuals engaged in logistics, entailing unnecessary duplication and delay in progress. This QUARTERLY has therefore been established to publish articles and other items of interest to logisticians, creating a medium through which the scientists can acquaint themselves with the problems of operational logistics, and through which the operating naval officers will be informed about the latest results and techniques devised by the scientists.



F. R. FURTH
Rear Admiral, USN

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## PREFACE

This is the first issue of a quarterly journal designed to draw together the ideas of all those interested in the problems and results of logistics research. The broad objective of the Quarterly is as follows:

> To provide a medium for the exchange of information and ideas on logistics and for the publication of research and expository papers, including those in certain areas of mathematics, statistics, and economics, for the purpose of increasing the efficiency and effectiveness of logistics operations.

The following policies have been set by the editors for conducting the Quarterly. Special emphasis will be placed on Naval logistics, but articles and discussions of Army, Air Force, national, and international logistics matters will be accepted. Articles on both the applied and the theoretical aspects of all levels of logistics will be included. Papers submitted for publication will be refereed for professional and scientific merit. Every cooperation with service and scientific publications will be sought. It is hoped that, within the limits of national security, there will be full freedom of expression and discussion together with objective and constructive criticism. The Quarterly will be unclassified, but classified supplements may be published if appropriate classified manuscripts are received. Although the Quarterly is published by the Office of Naval Research, it in no way represents an expression of official Navy opinion.

By covering both the applied and theoretical aspects, it is hoped that this Quarterly will act as a bridge linking the extremes represented by the work of the scientist, delving into basic theory and principle, on the one hand, and the ship Captain, concerned with his immediate problems of readiness for sustained combat, on the other. A wide variety of topics will be included, such as: broad economic theory, the technique of planning the replenishment of ships at sea, a discussion of a new wrinkle in a "Colonel Blotto" type game, or a discussion of the proper balance between the stock levels of various categories of supply required to capture an area by amphibious assault.

Logistics is both an art and a science, and the practitioner of the art will be severely handicapped until the basic science is more fully developed and understood. Until recently the emphasis has been on the techniques and on so-called "hardware," while the workings of the basic underlying principles have been merely surmised rather than clearly understood. Although this Quarterly will be devoted primarily to logistics research, it is recognized that the art and science of war includes strategy, tactics, and logistics, and that one cannot be studied without consideration being given the other two. For this reason, some of the articles may be concerned with matters which may not lie entirely in the field of logistics.

It is hoped that this Quarterly will stimulate discussion among the planners on the highest defense levels, the scientists engaged in logistics research, the military commanders, the officers controlling the flow of materials, the engineers developing weapons and equipment, and the manufacturers of military materials. It is believed that in this manner all parties concerned will find a common ground and will contribute to a better understanding of the underlying basic principles of logistics.

The Editors

Washington, D. C.
December, 1953

# LOGISTICS–WHAT IS IT? [1,2]

Rear Admiral Henry E. Eccles
U. S. Navy (Retired)

Since World War II the importance of logistics has been stressed at all levels of military activity, and military writers have discussed its various aspects at length. In many ways and forms it was a major matter of consideration in the unification of the armed forces. Today we understand it better than before the war, and therefore our organizations and plans are logistically superior to those of the past. Many of our most important unsolved problems are logistical, however, and there are still serious deficiencies in our logistic plans and capabilities. If we fail to correct these deficiencies, they may easily cause our defeat should we have to fight another global war.

While some of our present deficiencies are caused by our failure properly to apply what we already know, there are many areas where we are limited by our imperfect knowledge of the art and science of logistics. If we can perfect and apply this knowledge, it is probable, within our present economic limitations, that we can improve the effectiveness of our logistic support by fifty per cent.

Among the obstacles to improvement are the uncertainties that exist as to the meaning of the word itself and as to the proper place of logistics in military organizations and plans. This is because logistics has several distinct aspects and in each aspect the definitions and descriptions differ. Frequently, therefore, people may be talking from diverse points of view without realizing the effect this has on their descriptions and opinions. Each will be ascribing a different meaning to the word without realizing it.

In 1917 Lieutenant Colonel Cyrus Thorpe, U. S. Marine Corps, published an excellent little book, "Pure Logistics. The Science of War Preparation." This initial attempt to develop theory and principle apparently attracted little or no attention until five copies were found in the Naval War College Library in 1945. Some students of war wonder how many billions could have been saved had the significance of Colonel Thorpe's ideas been fully appreciated before 1941. Unfortunately the book is out of print, the publishing house out of business, and only few copies remain in the hands of individuals.

In his preface Colonel Thorpe says:

"The terms 'pure' and 'applied' may be used with the same meaning as to logistics as to other sciences. Pure Logistics is merely a scientific inquiry into the theory of logistics—its scope and function in the Science of War, with a broad outline of its organization. Applied Logistics rests upon the pure, and concerns itself, in accordance with general principles, with the detailed manner of dividing labor in the logistical field in the preparation for war and maintaining war during its duration."

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[1] This article appeared in the June 1953 issue of the United States Naval Institute Proceedings, copyright 1953 by the U. S. Naval Institute, and is being republished with permission of the Institute.

[2] This article was prepared under sponsorship of ONR Contract N7onr-41904.

It is the purpose of this article further to develop this approach and to discuss what is meant by the word logistics and what is its place in the naval establishment. It is not my intent to solve problems but rather to establish a background that will be helpful to those who are trying to solve them.

## PURE LOGISTICS

The word logistics is an abstract term which represents a very practical reality.

In its abstract sense it, like strategy, tactics, economics, and politics, is not susceptible to a single simple and permanent definition. Rather it can be described in a variety of ways in very general terms. If it is to be understood it must be approached from various points of view, and it must be discussed and described by reference to other intangibles and abstract terms. It is only through the consideration of one abstract term with relation to the other abstract terms with which it is naturally associated that the true picture can be developed and described.

Several of the most useful and enlightening descriptions of abstract logistics—or "pure logistics," as we may call it—are:

"Logistics is all that part of war which is not included in Strategy and Tactics." Col. Thorpe, "Pure Logistics."

"Strategy and Tactics provide the scheme for the conduct of military operations: Logistics provides the means therefore." Derived from Col. Thorpe, "Pure Logistics."

"The logistic process is, at one and the same time, the military element in the nation's economy and the economic element in its military operations." From a Munitions Board Study.

If we carry on the thoughts expressed in these descriptions we can see war itself in its abstract sense as comprising three interlocked abstractions—Strategy, Tactics and Logistics. If we draw on our experience and on history, we find that war is a matter of infinite variation, in which no two situations are precisely the same. We see that in all war situations the actions and decisions are based upon a blend of strategical, tactical, and logistical considerations. If we consider these abstract terms as three overlapping discs, we can see where some situations involve all three considerations, others involve two, and some only one. However, as we study, we realize that the dividing lines tend to blur and in some cases to vanish.

A somewhat different approach would be to say that, in its broadest sense, war is a combination of military, political, economic, and geographic situations and considerations. Here again we find a variable blend of abstract terms, each of which is subject to a variety of meanings and descriptions as well as to a variety of subdivisions; for example, geographic situations may well include climate and weather; political considerations can well include sociological aspects. However, military and economic considerations are related by logistics as indicated before, since it is "the military element in the nation's economy and the economic element in its military operations."

In order to fight a modern war we must "mobilize" our economic and industrial resources in order to create and support the necessary combat forces and to maintain our civil economy and health. Again we are speaking in abstract terms, for "mobilize" is a word that can be applied to many affairs.

Two descriptions taken from a Bureau of Supplies and Accounts paper are helpful in understanding this problem:

"Civil logistics is the mobilization of the civilian industrial economy to support the armed forces."

"Military logistics is the supplying of men and material, and the rendering of services, to the operating military forces."

If we now return to our earlier description and statement that "logistics provides the means for the conduct of military operations," we find ourselves on a relatively simple and broad abstract level using simple terms—terms that can be readily related and are adequate to cover the major problem of war. We also find that we have placed logistics on a coequal level with strategy and tactics, thus forming the three major subdivisions of the art and science of war.

If we are to practice an art, we must have a knowledge of the theories and principles of the underlying science. In this sense, principles can be considered as the statements of known cause-and-effect relationships. As of today these cause-and-effect relationships in logistics are virtually undeveloped. Patient, free research is required to develop these principles, and after they are developed they must be frequently re-examined to confirm their validity.

Someone may well ask—What is a logistic principle? An illustration is the "Principle of Priorities" which states:

"Priorities and allocations are required only in case of shortages.

"When shortages exist, both allocations and priorities are required to provide equitable distribution.

"Priorities used alone are self defeating and merely serve to worsen the defect they are supposed to correct."

This is an example of an imperfectly developed principle. In World War II this principle seemed to be borne out, particularly by the confusion in national distribution of scarce raw material and by the manner in which priority cargo waited shipment for months in air transportation depots. In the present partial war the evidence, while not yet wholly conclusive, tends to confirm the truth of this principle. If it be true, its application can greatly improve our efficiency in war.

## APPLIED LOGISTICS

Abstract speculations, theories, and principles have never prepared a nation to fight and have never won a war. All they have done is to enable man to understand his war problems and to assist him to solve them.

In order to prepare for war, we must define the practical tasks of the armed forces and we must assign these tasks to specific organizations and individuals. For this purpose we have organized the Department of Defense and the armed forces; for this purpose authoritative definitions have been published and specific tasks assigned.

The U. S. Joint Chiefs of Staff have defined logistics as:

"That part of the entire military activity which deals with: (1) design and development, acquisition, storage, movement, distribution, maintenance, evacuation and disposition of material; (2) induction, classification, training, assignment, welfare, movement, evacuation and separation of personnel; (3) acquisition or construction, maintenance, operation and disposition of facilities; and (4) acquisition or furnishing of services. It comprises both planning, including determination of requirements, and implementation."

This definition of applied or practical logistics is in no way out of harmony with the previous broad descriptions of pure logistics. Rather it amplifies them and reduces them to specific functional terms suitable for application throughout the armed forces.

Nowhere are there any orders as to how each service shall interpret this definition. This is wise, for conditions in each service vary. In the Navy Department logistics is rather loosely organized. The Deputy Chief of Naval Operations for Logistics has responsibility for coordination, and for the determination of requirements, while the technical bureaus have the operating function. In the Departments of the Army and the Air Force the Assistant Chief of Staff, G4 Logistics (Army) and Deputy Chief of Staff—Materiel (AF) have a much more direct authority.

The fact that a certain function is included within the defintion of logistics does not necessarily mean that the function in question is carried on by an organization that is logistical in name or is wholly devoted to logistical activity. (Failure to remember this important fact frequently causes confusion.)

However, it is of vital importance to understand one thing: that regardless of how the logistics functions are assigned and divided, the functions themselves are the same and must be performed by qualified officers. Furthermore, these functions must be supervised and coordinated by other senior officers; and these senior officers must not only understand the full implications of their responsibility, but also must understand the relationships involved in these tasks.

In the practical field, the definition of applied logistics, or the "What is it?," varies in accordance with the level of the organization being considered. But always logistics is concerned with "furnishing the means of war," which are material, men, facilities and services.

On the international and national levels, applied logistics deals with the broadest economic and industrial matters, such as: the sources and availabilities of raw materials; the state of the domestic economy and finances; the availability of manufacturing plants, skilled and unskilled labor, design and production engineers, management; and other similar affairs. Some persons may prefer to consider this as a combination of economic mobilization, industrial mobilization and military planning. The precise labels attached to the process are not as important as understanding the nature and inter-relationship of the functions performed.

The international process is exemplified by the military assistance activities and by the mutual security programs which have been placed under the administration of the Commander in Chief, U. S. European Command (U.S. CINCEUR) General Ridgway. The process is further illustrated by the work of the Screening and Costing Committee under General McNarney in Paris in 1951, and their immediate superior, the Temporary Council Committee, "The Three Wise Men." The Lisbon Conference of 1952 was almost wholly a high level logistic conference.

On this highest level the international and national situations and decisions must be continuously inter-related. Therefore, our own governmental organizations must work with their opposite numbers in other nations and with the special ad hoc or permanent international organizations. It is vital to seek harmony among the national and the international policies, strategic plans, and military programs. While it is naive to expect to achieve complete harmony, it is very important that we avoid contradiction. Policies and plans are made by both international and national agencies, but action is almost always by national agencies. There is an extraordinarily complex mixture of political, economic, strategic, and logistic factors.

At this top national-international level, the activities of the Joint Chiefs of Staff, the Munitions Board, the three military departments, and many other agencies and specially appointed individuals are so fluid and intertwined that they literally defy description. This condition emphasizes the importance of understanding the basic principles that operate in these fields.

Strictly national U. S. action follows a somewhat simpler pattern. The projects authorized and the funds appropriated by the Congress actually determine the logistic capabilities of the armed forces. Within the limits of these practical capabilities, the Secretary of Defense, the Joint Chiefs of Staff, and the military departments decide as to the specific forces to be built up and they formulate the broad strategic plans. The Munitions Board makes recommendations as to policies governing the allocation of both raw materials and finished products among the three services. This board also makes recommendations as to how the productive capacity of certain industrial plants should be allocated. The three military departments, through their technical bureaus and services, then actually procure and distribute to the operating forces the ships, planes, guns, men, equipment, supplies, and services which are the means of war.

The above are the major processes which constitute, or are associated with, the international and national level of logistics. It is sometimes convenient to call this the mobilization level or to consider it as part of the strategic phase of logistics.

The next level of activity may be described as the operating level or field level. It includes what the Army calls the Zone of the Interior and what the Navy calls the Continental Shore Establishment, and it takes in the theaters of operation, the fleets, the armies, and the air forces. It may be called "Operational Logistics" and may be said to include both strategical logistics and tactical logistics.

The logistics of the naval operating forces are based on the Sea Frontier, Naval Base, and Naval Depot systems, which channel the flow of logistic support and services to the fleets.

Planning for the actual conduct of this "Operational Logistics" is based upon the strategic plans and the broad logistics plans and policies of the theater and fleet commanders and upon their estimates of requirements. All of these furnish the necessary guidance to the type commanders who actually submit the requisitions and operate the basic logistic services afloat.

As we move from the theaters through the fleets to the task forces, we move from strategical logistics to tactical logistics; from the realm of long range plans and forecasts, to the actual repair and replenishment of combat forces. The techniques of tactical logistics are under constant scrutiny and improvement in actual practice. However, the techniques and procedures of so-called theater and fleet strategical logistics are frequently imperfect and sometimes neglected in peacetime.

Any one can understand the effect of a ship at sea running out of fuel and ammunition. But few officers understand the importance and nature of the long range concurrent strategic-operational logistic planning on theater and fleet level: the planning that will insure the readiness of task forces for sustained combat operations in time of sudden emergency.

In considering this division of logistics into various levels, we should always remember that each level overlaps with the other, both above and below. There can never be a sharp "chop line" of interest, although there are various "chop lines" of specific action responsibility. In all of the above stages and relationships we find changing general characteristics and emphasis.

On the highest level we deal in the broadest terms, and as we go down the chain we find ourselves being more specific. At first the emphasis is on civilian control, with important military participation. But as we go down to the operating level, the civilian interest tends to diminish and the military control increases. Civilian control tends to be strongest in the "producer" or business end of the logistic process; military control is strongest in "consumer logistics."

LOGISTICS IN PLANNING

There are two general types of planning in applied logistics: Logistics Planning and Planning for Logistics. Again, as in so many other fields, these overlap.

The first term, logistics planning, can be used to indicate the incorporation of logistics considerations in the development of strategic and tactical plans. It includes the determination of the basic logistic requirements and the general dispositions necessary to support these plans. All of this can be effectively done only by a constant and intimate relationship between the strategic planners and the logistic planners. In this relationship the former are constantly aware of the logistic capabilities and limitations, and the latter are constantly aware of the dispositions and employment being proposed for the forces, and of the estimated nature and strength of enemy opposition.

While the details of this Logistics Planning can be complex, the whole system rests on the following simple and straightforward fundamental sequence:

A proposed scheme of deployment and action and an estimate of opposition;

Determination of broad logistic requirements and determination as to which of these will be decisive or critical;

Determination of availabilities of these critical items;

Estimate of the effect of shortages upon the strategic and tactical courses of action;

Determination of what can be done logistically or tactically to alleviate these shortages or to overcome the handicaps imposed by them.

Since this process exemplifies the meaning of the expression "Logistics is a Command responsibility," it requires the complete understanding and active personal interest of the Commander himself, as well as the full participation of his strategic and logistic assistants.

The terms feasibility and calculated risk are frequently used in discussions of the problems of Command in War. These words have probably been clouded with more ignorance and superstition than any other terms in our war and postwar vocabulary. Each should be used with caution. They are closely related; yet each requires careful study and proper qualification in use. Final decision in either is a matter of the personal professional judgment of the Commander. In each case it is a question of how much risk and how much hardship the Commander is willing to impose on his subordinate forces and personnel in order to gain his objective. There are no absolute or arbitrary limits. The decision involves a process of selection of courses of action and of the development of plans that will make the most effective use of the combat forces and logistic resources that are available. This process is the highest test of military judgment and it requires close personal relationships among the commander and his responsible assistants.

The second term, planning for logistics, can be applied to the more detailed planning for the logistic support of the combat forces. Here is where the degree of flexiblity of the combat forces is almost wholly determined; it is the most "practical" type of functional logistics. Among other things it involves the details of the build up, distribution, and allocation of supplies, transportation lift, and repair facilities.

These two types of planning take place at all levels. The techniques may vary and in some instances they may telescope and become concurrent. Normally, however, the planning for logistics follows the basic decisions in the logistic planning stage. Since all of this planning requires close coordination between staffs and within staffs, we will now discuss staff organization as it applies to logistics.

## LOGISTICS ASPECTS OF STAFF ORGANIZATION

Since no standard staff organizations are prescribed, there is a wide variation in the manner in which various Commanders choose to organize their staffs. The organization is derived from the wartime tasks and responsibilities of the Commander. From the logistic point of view two general types of organization can be recognized.

The first type is found where the principal function of the Commander is logistics. The Service Force of a fleet is an illustration. The second is found where logistics is only one of several major tasks of the commander. Theater commanders and sometimes fleet commanders fall into this category.

In the first type, the Service Force, individual major functions—such as personnel, supply, maintenance and repair, medical, base development, plans, and operations—are usually assigned to separate staff divisions. Fuel, electronics and ammunition usually require special arrangements which vary according to circumstances. Since the whole task of the commander is a logistics task, the whole staff is a logistics staff. Therefore the Chief of Staff has the fleet logistic situation as his major concern. He, assisted by the plans officer, coordinates the entire logistics task of the command. This coordination is based on the strategic and the broad logistics guidance provided by the Fleet or Theater Commander. Under these circumstances there is no need for a separate logistics division on the Service Force Staff.

In the case of a theater or of a fleet staff, the situation may be quite different. Here it is important to group the major logistics functions under a single logistics officer who is the principal staff adviser to the Commander in logistics matters. During World War II this was done with notable success by Admiral Nimitz. However, this sound practice has not always been followed in our post war organizations. Instead, in peacetime, there has been a tendency to diffuse the logistic function throughout the staff. In a Theater or Fleet Staff, the Chief of Staff has many urgent concerns other than logistics. Therefore he cannot act as an effective logistics coordinator. If no single officer is given specific responsibility and authority for logistic planning and coordination within the staff, many loose ends or "holidays" develop. Under such conditions, it is probable that both the strategic and the logistics plans will be fatally defective. This can lead to disaster in war.

This need for centralizing logistics staff work in a single staff division does not imply that type and force commanders with major logistic responsibilities do not also act as advisers to the theater or fleet commander. There are two legitimate sources and channels, one the personal staff, the other the chain of command. Both are necessary.

Uncertainty and diffusion of logistic responsibility within naval staffs are illustrated in certain postwar staff organizations. In one instance the logistics section was established as a subsection of the material division. This is simply a case of reversing the cart and the horse: material is an essential part, but only one essential part, of the overall logistic problem.

In another case, planning for logistics support of naval aviation was assigned as a minor task of the operations division; the logistics division being aware, only by hearsay of aviation logistic requirements.

In staff organization, a common fault is to give the communications division cognizance over all matters dealing with electronics. This will inevitably break down in war because communications is only one aspect of the huge electronics problem.

Another fault is to attempt to maintain in the plans division a small logistics plans section to formulate the logistics plans. This results in waste of effort and poor plans, for if

the staff planning picture is to be kept clear, the logistics planning must be charged to the individual who is directly responsible to the Commander for advising and assisting him in logistics matters.

And finally there is the very human and understandable desire of each technical bureau to have its staff representative report directly to the Commander, without dealing through a "Logistics Officer" or through the Commander of a Logistic Force.

Many of the above variations in staff organization spring from the desire to operate with a minimum staff. Except for war planning, Theater and Fleet staffs in peacetime have relatively little to do, and what they do is relatively simple. Therefore, if logistic responsibility is diffused and scattered among several major staff divisions, such as plans, or operations, or communications, these divisions can probably handle certain current logistic matters with no undue effort and with no apparent harm. However, war instantly transforms this situation. The above divisions become swamped with their own primary duties and have no time for anything else. In fact they urgently require additional personnel to handle their extra load.

When war breaks out, fleet and theater logistics problems literally explode to huge size and great urgency. However, if the logistics responsibility has been diffused rather than concentrated, the logistics division will not be prepared to handle the emergency. Therefore confusion, serious trouble, and major waste ensue. These are the minimum results of the hasty improvisation which inevitably occurs. At the worst, a major military disaster can take place.

On the mobilization level as pointed out previously, the three services, the Joint Chiefs of Staff, and the Munitions Board are striving to relate war, mobilization, and budget plans to the national economy and to political factors. Here the relatively simple planning process and staff organizations previously described do not directly apply. The basic principles are the same, but the magnitude of the task and the realities of legal regulation and the budgetary and service competition require unwieldy organizations and a long drawn out planning process.

In this lengthy Planning Cycle, practically every major activity in the Department of Defense is involved, either directly or indirectly. For example, some people are working on budget and finance, some on ship design and construction, some on industrial plant capacity and allocation, some on manpower, some on raw material, some on transportation. Cut the cake any way you choose, call it by any name you want—Administration, Economic Mobilization, Industrial Mobilization, General Planning, or what you will—it is still the same reality. These men are working to "provide the Means of War in order to support the National Strategy." Regardless of the cut of the cake or the nomenclature used, it still must follow the basic logistic procedure. First determine your requirements; next figure out how and where you will procure what you want; and, finally, distribute it in accordance with the military needs of the situation. The consumers are the military; the producers are essentially civilian.

This process is logistics, logistics in a very practical sense. If those individuals doing this applied logistics have an understanding of the purpose, the relationships, and the principles of pure logistics, their work will have the coherence that is so essential. They can develop an efficiency and effectiveness impossible in the compartmentized operations of individual groups who see only their own day to day crises without relating them to the over-all problems and purpose.

## LOGISTICS AND ADMINISTRATION

There is some difference of opinion as to the relationship between the terms logistics and administration, since these terms have always been linked in military organization and planning. Where major staffs have used deputies to relieve the Chief of Staff of detail, it is usual to find a Deputy Chief of Staff for Plans and Operations and a Deputy Chief of Staff for Logistics and Administration. The Navy and the Air Force use a "Logistic Annex" to their operation plans, while the Army and Marine Corps use an "Administrative Order" to contain the logistic and administrative portions of their operation plan. This latter usage implies that pure logistics is a sub-category of the broader abstraction, Administration.

In one instance the term Administration has been described as:

"The management of all phases of military operations not directly involved in tactics, strategy and logistics."

This has been coupled with a secondary meaning:

"The interior management of units."

However, other authorities give administration an even wider scope and consider that it "denotes the endeavor of each member of an organization." In this sense it includes all organized planning, execution, and supervision of the planned action. Here it would apply equally to strategy, tactics, and logistics, and to all operations from all levels from the President of the United States down to the individual sailor performing his duties in peace or war.

Still another manner of treating the subject of administration is found in some staffs that group personnel and administration in one staff division. In some instances this staff division will have cognizance of legal matters, welfare, recreation, and postal affairs and military government, in addition to the distribution of personnel.

The term "staff administration" has a well recognized special meaning which usually includes the operation of the clerical and routing system of the staff, custody of registered publications and security, etc. It sometimes includes the supervision or operation of staff housekeeping and transportation facilities.

From the above we can conclude that Administration has such a broad general use and such a variety of special uses that, unless the context permits no misunderstanding, the term should always be clearly and appropriately qualified. It supplements but in no way replaces the term Logistics in military usage.

## BROAD OBJECTIVES

Sir Richard Livingstone once pointedly wrote:[3]

". . . by a mere technician I mean a man who understands everything about his job except its ultimate purpose and its place in the order of the universe."

One of the most important reasons for understanding the abstract relations between strategy, logistics, and tactics is to emphasize the purpose and the objectives of our effort.

The objective of logistic and administrative work is the creation and continued support of adequate combat forces in order that these forces may effectively support our national strategy.

The nature of modern war is such that, in order to fight effectively, there must be the greatest economy in the provision of these forces and their support. But economy is not the objective; it is merely one of the essential factors in the attainment of the objective. If the objective is jeopardized by over-emphasis of economy, fatal damage may be done to our national security.

[3] In "Some Tasks For Education," Oxford University Press, 1946.

If, on the other hand, the economic limitations of our national defense effort be not recognized, our national objectives and national security may be lost. Since the Russian development of Marxist doctrine has emphasized the employment of economic warfare and inflation to bankrupt countries they wish to take over or defeat, this latter danger must be recognized.

## CONCLUSION

In analyzing any subject it is desirable to reduce it to simple terms and to show them in logical relationship. In attempting to do this it is inevitable that statements be made which will appear obvious or trite and even repetitious to those who are experienced in the art. On the other hand, in attempting analysis in any complex field, it is likely that the pioneer must also use some relatively abstruse ideas and expressions. It is only after the exchange and interplay of the ideas of many enthusiasts that adequate clarification and simple statements of principles can evolve. With this in mind, the following thoughts are offered as to some of the most important facts and relationships which lead to an understanding of the question "What is logistics?"

In order to understand logistics one must understand both the word and the subject.

The word logistics is a symbol, an abstraction, and as such can be truly described in a variety of ways. It can be strictly defined only for a particular case at a particular time.

The subject of Logistics itself is a living, changing thing, it is not dead or static. Full understanding of the subject can come only by seeing it as a whole, by pondering upon its many elements and functions, and by practice in the art.

Pure logistics is an abstract term used to indicate the whole complex process whereby the means of war necessary to support a national strategy are determined, procured, and finally distributed to the combat commanders. Applied logistics represents the everyday practical application of this abstract process.

The objective of all logistic effort is the creation and sustained support of adequate combat forces. Economy is an essential factor in the attainment of this objective.

The applied logistics process of providing men, materials, facilities and services, comprises the performance of many specialized and technical functions. These functions include ship and aircraft design, construction, maintenance and repair; air and naval base design, construction, and operation; the operation of an intricate supply system; the provision of fuel, ordnance, and ammunition; transportation of all sorts; and personnel and medical services.

Some of these practical functions are performed by staff corps officers and line specialists, some by unrestricted line officers, and some by civilians.

Both pure and applied logistics can be roughly divided into two general classifications: mobilization logistics and operational logistics. Before operational logistics can function, there must be the prior performance of the mobilization logistics function: Mobilization logistics and operational logistics have a large area in common.

The nature of war and its component parts is such that sharp dividing lines cannot be drawn between strategy, tactics, and logistics. Instead, they blend and overlap in many continually varying ways. This is equally true whether one is thinking in abstract terms or in practical functional terms.

An understanding of both pure logistics and the broad aspects of applied logistics is essential to the exercise of high command.

Control of over-all applied logistics requires a knowledge of the problems of high command—particularly a knowledge of the relationships between the functional elements of applied logistics.

And finally, in any field of human activity, in the elementary and low level stage, individuals may have to be guided by specific rules and fixed procedures in rather narrow fields. As one gains in rank and in responsibility, the fields broaden and the rules and fixed procedures are gradually replaced by mature professional judgment based on experience and understanding of principles.

In this respect logistics in no way differs from any other art and science that challenges man.

* * *

# PRIORITIES IN THE NAVAL SUPPLY SYSTEM [1]

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## THE PRESENT SYSTEM

The purpose of any priority system is to provide a method of more efficiently utilizing scarce resources. That the Naval priority indicator is designed to perform this function for the Navy is shown by the following quotation:

> The priority indicator system is designed to provide a means whereby supply activities may process requests and ship the most important requirements first at such times as the capacity for processing and shipping may become limited.[2]

The two points of most importance in this statement are: first, the definite statement that the system is concerned with the shortage of what we may call indirect resources (processing and delivery resources of the system) rather than with shortages of the direct resources (end-use materials) and, second, the term "most important requirements." In the first part of this paper we shall devote ourselves to an examination of the problems raised by these different types of resource shortages and of means suitable for coping with them. We shall then proceed to a more detailed discussion of some of the particular features of the present system, and finally we hope to end up with some constructive suggestions concerning what are the desirable features of a priority system.

Before commenting in detail on the problems involved, it may be well to give a brief description of the present system as it is currently operating.[3]

The Naval priority indicator system has four classes of indicators labeled A, B, C and D, which may be roughly described as Emergency, Overhaul and Maintenance, Allowance and Interim, and Routine, respectively. The rules and guides for their proper application are summarized:

The A indicator is to be used for putting inoperative activities or installations into operation, for materials seriously required for health and hygiene, and for the support of special programs as designated by the Chief of Naval Operations (CNO).

The B indicator is to be used for maintenance, construction, or overhaul, for the carrying out of essential phases of operational tests and for allowance list materials required for overseas movements.

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[1] I should like to acknowledge my indebtedness to Professor O. Morgenstern of Princeton University for first indicating the problems in the fields of priorities and allocations. I must also express my appreciation to Mr. Kozak, Issue Control Branch Supervisor at NSD Bayonne, for the kindness he has shown in familiarizing me with the institutional background for this study. This study was supported by ONR Project N6onr-27009.

[2] BuSandA Manual, Vol. II, Supply Ashore, par. 23005, 1.a.

[3] The official description of the system may be found in BuSandA Manual, Vol. II, Supply Ashore, par. 23004, pp. 3-4 and 3-5.

The C indicator is to be used for research materials, shore construction, allowance lists of vessels, stock replenishment when current demand exceeds supply, and initial outfitting and preservation.

The D indicator is to be used for any request not falling into a higher priority classification but which is necessary for routine replacement or replenishment and for the filling of the on board allowances of inactive vessels.

In general, the assignment of priority indicators is to be made by the requesting activity. There are some general provisions for screening which are to be specifically determined by the Commander in Chief of the Atlantic or Pacific Fleet as applicable, or by the cognizant bureau or Supply Demand Control Point (SDCP) for shore activities. For some categories of materials the priority screening may be delegated down to the supply officer of the activity originating the request. This is true for such classes of materials as Clothing, Fuel, General Stores, Provisions, and Ship's Store Stock.

Other important features of the system are the necessity for a detailed written justification on requests with an A or B priority (except for the B priority authorized for allowance list materials for ships moving overseas). A realistic deadline delivery date (DDD) must be entered on all A, B, and C priority requests, and can be entered on D priority requests if this is felt necessary. It is explicitly stated in the manual that there is no direct relationship between the DDD and the priority indicator, the one being established purely to meet operational schedules and the other only to indicate the relative importance of the request in relation to other requests.

We may summarize the main points of the system as follows:

1. The application of the priority indicators is based on broad types of action except for the section authorizing CNO to grant A priorities to certain missions at his discretion.
2. There is a general decentralization of authority subject to the broad rules as given above, and with varying provisions for delegating the authority to screen the requests for priorities.
3. Deadline delivery dates are required on the higher priority requests and are to be established entirely independently of the priority indicator.
4. Explicit justification is required on all A and B priority requests with the exception noted above.

The system is a pure priority system in that it has no quantitative restrictions on the use of the priority indicators. (We will hereafter refer to preference schemes with a quantitative element as allocation schemes and preference systems without such quantitative elements as priority schemes.)

The system has a multi-component priority indicator rather than a single-component indicator. The priority indicator proper, the deadline delivery date, and the justification are all given on the request document and all play a role in the determination of the order of processing. When a request comes into a supply depot or center, the order of processing is determined by the Editing and Scheduling Section of the Issue Control Branch. The editor, by looking at the three components of the priority as mentioned above, determines by a process of judgment whether the request will be processed as an Emergency, Expeditor, or Routine request.

The Emergency requests are carried through the system by hand and the issue is made on the requisition itself rather than waiting for the cutting of an invoice. The Expeditor

requests are also carried through the system by hand, but go through all of the normal processes, including the cutting of an invoice. The Routine documents follow the normal flow patterns as set up, waiting for scheduled pick ups, etc.

To a great extent then, the effective priority is not the priority indicator itself but rather the type of treatment the document receives within the depot. This classification is made by weighing the deadline delivery date, the priority, and the justification. The manual states that it is entirely possible that an early DDD with a low priority indicator be processed and delivered before a late DDD high priority indicator request, if such processing will not interfere with the scheduled delivery of the high priority material. The effective priority is determined by a process of translation from the multi-component priority indicator as received by the depot to the classification of the processing of the document.

How is this translation effected? The first element taken into account is the DDD. A C-priority document with a strong justification may be treated as an Emergency request after consideration of the geographical distance of the requesting activity and other elements affecting the time of shipping, the composition of the current workload with respect to priority, and other factors. If there are few A's moving through the system, then more B's and C's may be given the Emergency or Expeditor treatment. We may say with certainty, however, that the priority indicator proper is not the only, or even the predominating, influence going into the determination of the type of treatment which a request document will receive. The translation depends on the judgment of the personnel at the depots as they balance the various factors going to make up the entire priority picture of each request document. For many of the requests, of course, there is no problem. But for the borderline cases there is no precise and unambiguous method of determination.

Another important feature of a pure priority system is the chronological element. Request documents arrive at the depots in batches over time. If a routine requisition takes about seven days to work through a Naval supply depot, counting from the time a requisition is received to the time the goods leave the depot, it is clear that any requisition which has been already started through the system has absolute preference over any requisition which has been received seven or more days later. The time, in fact, is even less than the full processing time because of the following procedure within the depot. As soon as the requisition leaves the Issue Control Branch it goes to the Stock Control Branch where a stock review clerk checks the items on the request against the appropriate stock status card. If the materials are shown on the cards as being on hand, the items are expended. As soon as this is done the stock status cards reflect the stock change. If the item expended was the last item in stock, an A-priority request following the original request cannot be satisfied and must be referred to a higher echelon of supply. To set up a system whereby the higher priority requests could overtake the lower priority of previously received requests and claim the material from them would entail the setting up of a very expensive information system within the depot.[4]

## LIMITATIONS OF PURE PRIORITY SYSTEMS

We now proceed to examine several features of simple priority systems which constitute rather severe restrictions on the usefulness of such systems. If we take the stated purpose

[4]While there is a system of material reservations within the depots, it was not designed to mesh with the priority indicator system. Material is reserved for specified advanced bases, and there is no method of differentiation in priorities which would allow higher priorities to claim the reserved material.

of the Naval priority system in particular, as given, a question which naturally arises is, "Does the present system serve to accomplish these designated ends?" As we shall see, the answer is probably "No." The Naval supply system is composed of a large number of resources combined in various ways to form supply depots, centers, SDCP's, and so forth. This gives rise to two problems. First, the Navy must make sure that the supply system is not too large for what it is asked to do. Too large a system results in idle resources involving costs with no return to the Navy. Important and fundamental as this problem is, since it does not concern priorities, it will not be discussed here. A second and separate problem arises when the indirect resources of the system are faced with a workload which is more than they can handle. It is this problem which the priority system is designed to solve. The problem is one of making sure that the tasks are performed in order of importance so that if there are more jobs than can be done, the ones left undone are the least urgent. The priority system is designed to accomplish this by a method of ranking the requests in order of importance.

The evidence would seem to indicate that the present emphasis is wrongly placed. There are two arguments to support this position, both equally cogent. The first (and perhaps more immediately practical) argument concerns the most likely type of shortage. Historical experience as well as certain logical considerations seem to point to the fact that the first pressures on any modern military supply system, which is not grossly understaffed, will be felt in the area of shortages of particular materials, rather than of inadequate processing and delivery facilities. If we consider the tremendous number of line items in the Naval supply system and the fact that a satisfactory theory of inventory control has yet to be attained, we can conclude that we are presently unable practically and in principle to maintain the proper amount of every item on hand.[5] The experience of the alleged ammunition shortage during the Korean campaign, although never completely clarified, does indicate the potential seriousness of such shortages. Furthermore, for various reasons, many of the items carried by the services are currently in short supply, even when most of the supply activities are operating well within their capacities. If materials shortage is the first critical area in the supply system we must re-examine the present system in this light. Since the system was not designed for the most likely set of contingencies, i.e., material shortages, it can be defended only if it functions with equal efficiency under conditions of shortages of direct or indirect resources.

This brings us to our second argument. We begin with a fairly obvious tautology which will in spite of its almost trivial nature prove to be useful. The utilization of indirect resources implies that these resources are expended in furthering some part of the process of material delivery. Therefore, if some material has been delivered to a place or at a time where its use was less than optimal, somewhere in the supply system resources have been expended in less than an optimal manner, i.e., those indirect resources which were used to effect the less than optimal delivery. The converse of this statement is equally true; when any indirect resource is expended in some other than the most efficient use, some material item must necessarily be delivered at a time or place which is not optimal. These rather truistic statements do indicate the equivalence of optimality of the expenditure of direct and indirect resources within the supply system. The problems of most efficient utilization of indirect resources and direct resources are seen to be one problem, although the solution of this problem can be tackled with emphasis on either the direct resources or the indirect resources. If we

[5]For a discussion of the inventory problem see T. M. Whitin, "Military Inventory Control," Research Reviews (ONR), December 1952, pp. 1-7. A fuller discussion of a more general nature may be found in the same author's Theory of Inventory Management, Princeton University Press, Princeton, 1953.

can now demonstrate that there must be a quantitative element in the solution of the problem of allocating the direct resources of the system, we will have shown that any system of (non-quantitative) priorities is inadequate for the task of controlling resource utilization, whether one approaches the problem from the side of direct or indirect resources.

The preference which is accorded to any particular want is, in general, not absolute. The want which is originally the most urgent will become less urgent as goods are provided to satisfy that want. We note here the important qualifications on this statement which is applicable only to wants which are divisible in terms of the goods required for their satisfaction, and to wants which can be completely ordered. In the majority of cases, as more goods are directed towards the satisfactions of any particular need a point will be achieved where the original becomes less urgent than some other want. It is at this point that resources must be diverted from the first to the second use if maximum utility is to be attained. This principle which is generally accepted by economists points up the necessity of the quantitative considerations in this area.

It is not deductive processes alone which point to the superiority of quantitative techniques of controlling resource flows. The lessons of experience also point the same way. This is true not only of the relatively simple Naval supply problem but also for the vastly more complex situations which arise in the national economy. One of the histories of our industrial war experience notes:

> The granting of preference ratings on an individual basis is essentially a device for providing assistance rather than control. It is a means of identifying the more urgent requirements and giving them a preferred status in procuring production materials. This works until the urgent requirements begin to compete with one another on a time basis. The principal factor hastening the breakdown of the individual rating procedure was the absence of any quantitative measure of priority activity.[6]

In most countries as well as in the United States during World War II, as supplies became more and more stringent, the simple priority system had to give way to some method of allocation.

> A quotation closer to military interests also illustrates this point:
> In the rare exceptions that can be cited of Priorities without Allocations working well, a closer examination will probably show that there was no scarcity of the commodity at all; therefore, controls or priorities were unnecessary to begin with.[7]

We conclude that allocating direct resources or materials in the Naval supply system would be more efficient than a system of priorities. Also, proper utilization of the indirect factors would be insured automatically in the process of allocating the materials. This could also be done in principle by proper allocation of indirect resources, a task which is practically impossible for the present because the entire supply of the indirect resources is not known in terms which are significant for the ends the allocations are designed to further. We would have to know the capacity of the Naval supply depots and centers before we could allocate the use of these resources. Although the capacity of the system, or of any particular

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[6] D. Novick, M. Anshen, and W. C. Truppner, Wartime Production Controls, Columbia University Press, New York, 1949, pp. 47-48

[7] H. E. Eccles, "Principle of Priorities," unpublished paper, July 1, 1953. From the context we can easily conclude that the author is referring to scarcity of the indirect resources as well as the "commodity."

depot, is possible to ascertain, in principle, our knowledge does not yet extend this far. Preliminary results obtained in the potential capacity study conducted by the Naval Supply Research and Development Facility at Bayonne in consultation with the Economics Research Project of Princeton University seem to hold some promise for the eventual acquisition of such knowledge. Allocations is, of course, only one of the possible uses of such knowledge of capacity of indirect resources. This matter need not concern us, since we can, by a suitable method of allocating the direct resources, assure ourselves that the indirect resources are being used in an efficient manner.

We may recapitulate the second argument as follows: Optimal distribution of materials is achieved if, and only if, the indirect resources are used optimally. An optimum distribution of materials can be achieved only through some quantitative system of allocation. Therefore, to insure that all direct and indirect resources are used in the most efficient manner, some quantitative element must be introduced into the priority scheme. The quantitative allocation of materials is necessary and sufficient for the insuring of the optimal utilization of all resources.

## SPECIFIC CRITICISMS

We now turn to some more specific criticism of features of the present system which are peculiar to the system itself and which are not attributes of priority systems in general. We shall then turn to some constructive suggestions concerning the desirable features of a method which would use supply system resources most efficiently.

Our main criticisms of the present system refer to the rules which indicate the circumstances under which the various priority indicators are authorized for particular request documents. The categories refer to these types of action rather than to authorizing higher priorities for those actions which are operationally most significant regardless of type. For instance, it cannot always be true that it is more important to put an inoperative installation into operation than it is to fill allowance lists for ships which are moving overseas. Nor can it always be true that this latter type of action is more important than the filling of the on board allowance of any active vessel. Yet the three types of actions listed above are entitled in all cases to A, B, and C priorities respectively. The only statement in the rules which even hints of the fact that particular operations are a more significant concept in the Navy for purposes of supply than types of actions is the provision whereby the CNO is authorized to assign A priorities to special programs.

Even if we agree with the present method of classification the system still needs some overhaul of the rules to attain more fineness of discrimination among requests. We have mentioned above that the DDD carried much weight in the translation of the priority components of the request documents into the effective priority treatment within the depot. The table given just below shows that for a very large percentage of request documents it must certainly be something other than the priority indicator which serves as a guide to the depot personnel in the ordering of processing. About three-quarters of the requests received at Bayonne during the time period covered were of the C-priority classification. It is indeed hard to believe that of the large numbers of C priorities there were not substantial differences in urgency among them. It is also hard to believe that the DDD itself was a sufficient additional guide to the personnel scheduling these requests. We suggest therefore that there is room for refining the system as well as making the authorizations classification fall into a more significant breakdown. This will be discussed below.

Request Priorities, NSD Bayonne
Dec. 25, 1949, to Dec. 25, 1951

| Priority Indicator | Number (in thousands) | Percent |
|---|---|---|
| A | 32 | 1.7 |
| B | 380 | 20.2 |
| C | 1,379 | 73.2 |
| D | 93 | 4.9 |

It seems likely that there are two opposing forces at work in any system which attempts to achieve an efficient utilization of resources and which is operating within the framework of a large and complex structure. One is the fact that a simple system is not likely to provide solutions to complex problems. The other is that as the system grows in its ability to handle complicated situations, the administrative overhead rapidly becomes more burdensome. The problem, then, is to balance the advantages gained from better performance of task against the disadvantages which accrue from administrative complexity. Neither of these forces can be considered alone, although in the design of the present system one might almost be led to believe that simplicity was the only criterion. As the table above indicates, the degree of discrimination is insufficient for differentiating requests with good judgment concerning the ordering of the processing.

Secondly, we note the very large degree of decentralization of the actual assignment of priority indicators, subject to the broad rules which were paraphrased above. While there is some provision for the screening of the requests, it is left very much up to the discretion of the various commands. If there is any material shortage, the only way in which the total stock of the limited materials can best be divided among their various uses is by some organization[8] familiar with the total available amounts of the scarce materials and also with all of the possible uses to which these materials might be put. This implies that the present decentralization may be a serious fault of the system in the event of a material shortage. It may lead to waste and, even worse, to the nondelivery of necessary items.

## SOME SUGGESTIONS FOR A SYSTEM OF ALLOCATIONS

The most important set of actions which the Navy takes are its assigned missions, which in peacetime may be of a primarily training nature and in periods of war are integral parts of the over-all military strategy. These missions themselves vary in importance but presumably every element of the Naval organization is contributing to one or more of them. The rough outline of a system of allocations which follows is based primarily on this idea that the most important concept in Naval logistic thinking is the accomplishment of mission. If this idea is accepted, it follows that any supply system must be mission-slanted in design and outlook. We must then look for a method of setting up a system which insures that the supply resources are allocated to missions rather than to broad types of action. This may be done in many ways. Perhaps the most feasible plan is some method of assigning priorities to missions.[9]

[8]The size and complexity of the task may call for a somewhat complicated and fairly large organization. Presumably, this organization would have to work in close conjunction with the various SDCP's as presently set up to keep in touch with available supplies and would also have to work closely with the higher commands in order to know the requirements. The setting up of this organization should, however, be studied now and its feasibility determined. In the event of an all-out war, such central control of materials would be imperative.

[9]Note that the assignment of priorities must be extended throughout the whole system, to include shipping as well as intra-depot resources.

These priorities should be assigned by the operational command which is fully informed of the relative operational importance of the various active units of the Navy. When the operational commander has been informed of the priority which applies to the forces under his command, then some standard method should be set up whereby he can screen all of the requests originating in his command to determine if they are necessary and urgent enough so that the highest priority may be assigned to them. It is at this point also that the requested DDD should be evaluated in the determination of the priority to be assigned rather than at the depot level. The commander in the field would seem to be better equipped to evaluate the request by taking into consideration the justification and the requested DDD and, on the basis of these factors, to assign a priority indicator which would in itself be the net result of the various components of the original request. This process would probably entail the utilization of more than four categories of priority. This in itself would be desirable for the reasons pointed out above relating to the degree of discrimination of the present system.

Coupled with the field screening of requests and the centralized (by mission) assignment of priority indicators would be some method of priority reservations of any materials which might be in short supply. The Bureau of Supplies and Accounts should issue periodic lists of those materials which can be issued only on a given priority or higher. This would amount to the rationing of the goods to those missions considered most important. The determination should be made on the basis of usage factors plus the estimated supply. If sufficiently good usage factors ever become available, then this system would amount to a system of allocations in the following manner. If the proper data were available to the Naval logistic planner, the planner would know within some well-defined confidence limits what would be required by each mission. Knowing this and the amounts of material on hand and on order, he could make a quantitative allocation to each mission, according to its importance. Because no usage tables can predict needs exactly, he would probably not allocate all materials but would leave a safety allowance unallocated. Then the higher priority missions as designated by the highest echelons of command could, in the event of unforeseen contingencies, use these higher priorities to claim items from the unallocated stocks. Thus the priorities would serve the double function of, first, guiding the supply personnel in the processing of the request and, second, authorizing claims on materials in excess of the usage tables.

The supply depot under this system would have no translation to perform. The highest priority indicators would automatically be processed as Emergency documents, the second group of priority indicators would automatically be treated as Expeditor, and the remaining requests would be processed as Routine in order of priority indicator. For example, if we had seven separate priority indicators, the first two might possibly be always treated as Emergency, the third and fourth highest might always be treated as Expeditor and the remaining as Routine, in order of priority. The exact handling would, of course, be a matter to be worked out as experience with the system was attained.

The solution outlined here transfers the responsibility of assigning the priorities from the individual requesting activities themselves to the next higher echelon of command. A reasonable compromise places this responsibility in the hands of the mission commanders because it both represents a workable step towards the necessary centralization and places the decision-making function in the hands of the personnel best equipped to make the necessary judgments. The assigning of priorities to short and supporting activities should also be made in accordance with the missions which these activities support rather than by any over-all pattern of types of activities.

We realize that the foregoing outline of a system of allocations is rough and perhaps raises as many problems as it attempts to solve. Nonetheless, we believe that in the main the advantages of a quantitative centralized system of controlling the resource flows are too weighty to overlook.

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# ON THE SPAN OF CENTRAL DIRECTION[1]

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Although the problem of centralization has received (and is receiving) much attention, an adequate definition of centralization has not yet been devised. People frequently have preconceptions concerning whether centralization is good or bad, although there is often a complete lack of reasons for these opinions.

Given any of the typical definitions, it is a simple matter to construct paradoxical examples that immediately demonstrate their inadequacy. For example, in the current controversy concerning unification of supply, much of the discussion implies that unification is equivalent to centralization. This is by no means necessarily the case. It is quite possible to have a unified but extremely decentralized supply system,[2] and conversely to have a separate supply system for each service with a high degree of centralized control of each service.

The problem of military supply is a general equilibrium problem. In wartime, more goods can be produced for the military only at the expense of giving up production for civilian sectors. Within the armed services, more tanks can be acquired only at the expense of ships, planes, etc. Unless some authority is placed over the services, grave difficulties are likely to arise concerning priorities, and it is not likely that an optimal program will be approximated. Nor was a system of priorities determined by the former Munitions Board, whose members owed allegiance to separate services, satisfactory. A recent book by the War Records Section of the Bureau of the Budget contained the following passage:[3]

> During the summer and fall of 1942, scores if not hundreds of production lines were closed down for brief periods when the flow of materials ceased, and the conflict of high military priorities threatened to strangle the entire war production program, military and civilian.

A conversation with Archibald Alexander, former Under-Secretary of the Army, stressed some of the difficulties that can arise because of different profit rates allowed manufacturers by the three services. Since manufacturers give preference to the service allowing the higher profit percent, the working of the priority system is greatly impaired. Clearly, there is a great need for a new priority system that gives more adequate consideration to the over-all viewpoint.

Nevertheless, the conflict between the need for realism and detail on the one hand and for coordination on the other will persist. Ely Devons, wartime director of the Ministry of Air Production in England wrote that:[4]

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[1]The research for this article was done at Princeton University under Office of Naval Research Contract N6onr-27009. This article is a revision of a talk given at the Fourth Annual Logistics Conference held in March 1953 at the George Washington University.

[2]Admiral H. E. Eccles is currently working on clarification of the semantic distinction between "unification" and "merger."

[3]The United States at War, p. 280.

[4]Ely Devons, Planning in Practice, p. 14.

> Every attempt at planning reveals these two problems: first, the need to split up the field to be covered so that each administrative unit can deal efficiently with its own sector; and second, the need to secure that the actions of these separate units all fit into the general plan. But the implementation of these principles always leads to a conflict. For the first requires delegation and devolution, so that plans can be manageable and realistic; and the second requires centralization, so that plans can be coordinated.

Returning to the problem of defining centralization and decentralization, typical definitions state that under a centralized management the important decisions are concentrated at one central point. This type of definition clearly ignores a point brought out by Herbert Simon, namely, that there are two very different aspects to centralization, "...decision-making powers may be centralized by using general rules to limit the discretion of the subordinate. On the other hand, decision-making powers may be centralized by taking out of the hands of the subordinates the actual decision-making function."[5] In a sense, the latter type of centralization is a special case of the former where the rules lead to a unique decision. An organization might be changed from completely "centralized" to completely "decentralized" by nominal changes in the decision-making process.

This illustration indicates some of the difficulties inherent in defining centralization. A somewhat different type of problem that may arise may be illustrated by the recent history of a large mail order company. The company was considering a decentralization of certain functions that had previously been highly centralized. Specifically, the company was considering discontinuing the keeping of sales records for style goods in a central office, the central office review of purchase orders, and the central office revision of demand estimates. At the same time the company was engaged in bringing together under one supervisor certain retail and mail order operations that had been handled by different supervisors. Purchasing and disposal of surplus are among the functions thus centralized. Clearly this example is centralization in one sense and decentralization in another. Is the net result more centralization or less? The answer is not known. There is no single measurement that is satisfactory for measuring the degree of centralization. As soon as more than one characteristic must be measured, the problem of assessing the relative importance of the various characteristics arises.

If we assume for the moment that we can measure the degree of centralization, then the problem of ascertaining the optimum amount arises. Here, again, many factors are relevant such as the size and nature of the organization, the capabilities of its members, and the costs of information. We could take the usual approach of writers in the field of Public Administration and list various advantages and disadvantages of centralization. But unless there are methods of evaluating the factors involved, this approach is not very fruitful. The problem of assigning values to the relevant variables is the most urgent and the most difficult task in the field.

As just indicated, much literature in the Public Administration field consists of listing advantages and disadvantages. Other literature in the field consists of a study of special cases. Herbert Simon wrote of the "steady shift of emphasis from the 'principles of administration' themselves to a study of the conditions under which competing principles are respectively applicable."[6] This is accomplished, of course, by choosing situations where the weights

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[5]Herbert Simon, Administrative Behavior, New York, 1947, p. 234.
[6]Ibid., p. 240.

attached to some particular aspect or aspects is sufficiently high to dominate the other aspects. One leading authority in the field has written the following on integration:[7]

> Public Administration has only a loose content of incomplete observations, speculative hypotheses, and unwarranted conclusions to offer the student and administrator who wishes to inform himself about the best settled experience.

Other quotations indicate that the problem of organization requires analysis, balancing, and setting values, but do not go on to carry out the analysis. In this paper, several aspects of the centralization problem that lend themselves to formalization will be presented. The models are partial equilibrium models and are at best only approximations to a complex reality involving all the various effects and their interactions. It is possible, however, in the state of present knowledge, that a systematic evaluation of these partial effects may be useful, for it may frequently be the case that indirect repercussions are small. The analysis is primarily intended as an example of the type of analysis that may be of use in many concrete situations involving a choice between centralization and decentralization, or rather between a higher and a lower degree of centralization. The models discussed, as most economic models, involve a balancing of the advantages and disadvantages of centralization from various specific viewpoints.

Before discussing the aspects of the problem that can be formalized, I shall make a few general remarks on possible limits of centralization. It is generally agreed that increases in the amount of centralized control bring about substantial increases in the difficulty of the task of coordination. For example, Herbert Simon mentioned that "referral upward introduces new money and time costs into the decision-making process."[8] Also "noise" is introduced that interferes with the transmission of information, thus necessitating the making of decisions on the basis of worse information than that available at the lower level. The resulting high-level decision may therefore not be better than the lower-level decision. Even in the event that better quality of decision prevails at the high level, the benefits must be weighed against the concomitant costs.

Urwick has written that there is no question that the functional method of distributing responsibilities does "very much increase the difficulties of coordination, and that, generally speaking, insufficient thought and attention have been given to the problems which it raises in this direction."[9] Similarly, Russell Robb wrote, "The returns from increasing organization do not continue proportional to the effort and limits are reached ... the loss on account of difficulties of coordination sometimes more than counterbalances any advantage from economy that can come from functional division."[10] One type of extreme view of the difficulties inherent in central coordination is the following:[11]

> The central planning of production ... in the vigorous and historically not unwarranted sense of the term ... is strictly impossible; the reason being that the number of relations requiring adjustment per unit of time for the functioning of an economic system of n productive units is n times greater than can be adjusted by subordinating the units to a central authority.

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[7] Earl Latham, The Federal Field Service, Public Administration Service, Chicago, 1947, p. 6.
[8] Op. cit., p. 236.
[9] L. Urwick, The Elements of Administration, New York, 1944, p. 62.
[10] Russell Robb, "The Limits of Organization," reprinted in Papers on Organization and Management, edited by Catheryn Seckler-Hudson, American University Press, 1946, pp. 156, 171.
[11] Michael Polanyi, The Logic of Liberty, Chicago, 1951, p. 111.

Such statements, in the absence of a fairly detailed specification of the type of relationships involved, are almost devoid of worthwhile content. They are a futile attempt to introduce a factitious precision into their analysis.

Difficulties of coordination either explicitly or implicitly play an important role in the "decreasing returns to management" that has received much emphasis in economic theory. Some of these difficulties were spelled out in detail in Sir Oliver Franks' Central Planning and Control in War and Peace,[12] as the following passages indicate:

> All really large-scale organizations have tendencies to inefficiency.... Successful determination of policy requires high intellectual ability to hold the many diverse factors clearly before the mind and grasp firmly their importance and their relationships.... Abilities sufficient to cope successfully with the planning of a small-scale organization are insufficient to deal with the same thing on a larger scale.... Inertia has been described as the characteristic vice of the bureaucrat.... Really large-scale organization compels men to take many decisions at a point remote from the places where their decisions will take effect. They work on paper. The in-tray and out-tray symbolize the situation. It is not easy to see beyond the paper and hard to preserve a lively awareness of the real issues.... Large-scale organization tends to produce a steady flow of orders, instructions, and memoranda from the centre. This has two results, one direct and the other indirect. The first is that initiative at the periphery is restricted progressively as the authority of the centre is asserted.... In the second place a habit of mind is engendered in the staff of the organization. They come increasingly, when confronted by the need to act, to look for rules and precedents in the light of which to determine action rather than to seize on the situation for what it is or attempt to devise techniques and methods of action which will yield a fully satisfactory outcome from it.

In a sense, a production line constitutes a highly centralized entity. There is need for extremely close coordination between all the components of the final product. Disturbances in one part of the line may necessitate stopping production. The operating costs of coordination in this extremely centralized case, however, are not high, once the production line has been set up. This type of extreme centralization can take place only in a special technological set-up. In fact, the degree of centralization should not be considered independent of technology. Consideration of the decision-making process during battles may illustrate the importance of technological considerations. In early days, the supreme commander accompanied his men to the battle field and made key decisions on the spot. In recent wars, commanders have found it necessary to delegate important decision-making powers to the field. The latest developments in communications, television, etc., may make possible a recentralization of decision-making, for rapid access to good information may enable coordination to be brought about efficiently from the center.

Difficulties of coordination constitute one of the principal limiting factors to centralization of many types of organization. Ely Devons stressed this factor, stating that the effective limiting factor was "the limitation in the number of variables that could be comprehended by one brain ... the decision had to be taken on the basis of rough-and-ready orders of magnitude that the coordinators carried in their minds."[13] It may be the case that, as a production line

[12] Harvard University Press, 1947, pp. 27-30.

[13] "The Problem of Co-ordination in Aircraft Production," in D. N. Chester, Lessons of the British War Economy, Ch. VII.

facilitated better coordination in the above case of centralization, modern techniques in large-scale computation may effect better coordination in over-all planning problems. If the limiting factor is the number of interrelationships that can be handled in one brain, it is evident that an electronic brain can handle more of these than can a human brain. Modern programming techniques may thus be a tremendous aid in military planning. Programming estimates of feasible production plans may be far from accurate—and yet, if they are better than the estimates they replace, they constitute an improvement. The evaluation of these techniques depends on much more empirical testing than has been completed at the present time.

Some factors limiting the degree of centralization have been discussed. Current literature in economics often stresses these limits in discussions concerning decreasing returns to scale and their causes. At the same time, however, some factors operate to effect increasing returns to scale. The lack of emphasis on these factors constitutes an important deficiency in economic theory. The facts that large-scale industry makes possible a high degree of specialization and sometimes enables better adjustment of indivisible factors have received some attention, but certain other factors that bring about increasing returns have been discussed hardly at all. For example, the "repairman problem" cited by W. Feller[14] is certainly relevant for economics. This problem is concerned with automatic machines which ordinarily require no human care. However, they are subject to breakdown and call for service. When the number of breakdowns and the repair time involved are assumed to be random variables, the optimum number of repairmen can be computed. The results of this analysis have been successfully applied in Swedish industry. The number of repairmen increases much less than proportionately with the number of machines if the degree of protection against stoppage is maintained constant.

Another example is that of group feeding, where the amount of food required per person to provide protection against random variations in demand increases much less than proportionately with the number fed in a mess hall. Still other examples occur in the field of inventory control, both with respect to purchase quantities and safety allowances. Economies of centralization will appear in the case of safety allowances wherever demand is subject to random variations; in the case of purchase quantities, economies of centralization may appear even in the cases of known demand. These cases will be discussed below, as they play an important role in the partial equilibrium analysis models constructed. In the field of inventory control empirical observations are in agreement with our theory. For example, W. F. Luttrell wrote of slower turnover rates caused by the existence of branches.[15] Other empirical evidence is not hard to find.[16] A final economy of scale is that lower cash reserves are required for liquidity purposes, relative to the volume of sales. In general, whenever there are random variations in economic variables, it is quite likely that economies of scale exist. Since random variations permeate a great many aspects of economics, these economies are of considerable importance.

Now, returning to the analysis mentioned above, four models will be discussed, named for convenience (1) the decision-making model, (2) the safety-allowance model, (3) the location

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[14] C. Palm, "The Distribution of Repairmen in Servicing Automatic Machines," Indistritidningen Norden, Vol. 75 (1947), pp. 75-80, 90-94, cited in W. Feller, An Introduction to Probability Theory, New York, 1951, p. 379 n.

[15] W. F. Luttrell, The Cost of Industrial Movement, Cambridge, 1952, p. 19.

[16] T. M. Whitin, "Inventory Control in Theory and Practice," Quarterly Journal of Economics, November 1952, pp. 502-521.

model, and (4) the purchase-quantity model. It should be emphasized that these models are not denying the importance of noneconomic factors. At the same time, it should be made clear that there always remains an economic problem along with the others. For example, if a certain degree of security is a specified objective, the economic problem is to minimize the costs involved in attaining it. It is with the economic aspect of the models that the following models are concerned.

In the decision-making model, assume the existence of N independent firms; assume secondly that the nature of demand interdependence between firms is known, as are the costs of transmitting information concerning demand to an information center, the costs of operating on the information, and the costs of feeding back improved estimates of future demand to the firms. In the case of random demand variations, the demand feedback is subject to decreasing returns. Since the costs remain constant, it is profitable for only n of the N firms to transmit information to the information center which would feed back the results to all N units. The greater the demand variance, the larger is the number of firms that should submit sales results to the center. In the event that demand is certainly known, and there are no systematic interrelationships, then the need for an information center disappears and each firm or supply center makes its decisions independently.

Realistic examples that possess some of the essential characteristics of the model are not hard to find. For example, in the mail order business, it is often contended that there is more reason for centralization of control of style goods than for staple goods because of the volatility of demand. The centralized controller feeds back sales estimates to the individual mail order stores, based on the sales history of all the stores.

In the safety-allowance model, assume the existence of various retail outlets. Assume that supplies can be distributed by several alternative methods, among which are:

(a) Directly from manufacturer to the outlets, with no inter-outlet transfer.

(b) From manufacturer to a central storage warehouse and from there to the retail outlets.

The following assumptions are made:

(a) The demand distribution for each outlet is known.

(b) The following delivery times are known: $t_{mr}$, the time from manufacturer to retail outlets; $t_{mw}$, the time from manufacturer to warehouse; and $t_{wr}$, the time from warehouse to retail outlets. These times are the time elapsing between sending the order and receiving the merchandise.

(c) The costs of depletion are known.

(d) Carrying charges are known.

For different values of the above parameters, one can construct some situations where goods should be shipped directly from manufacturer to retailer and other situations where goods should be shipped to retailers via a central warehouse. For example, the larger the demand variance, the more likely, other things being equal, it is that the central storage function of the warehouse is desirable.[17] This results from the fact that safety allowances increase less than proportionately with demand; hence, central storage reduces carrying charges. Similarly, other indications that central warehousing will lead to reduction in costs are a large difference between $t_{mr}$ and $t_{wr}$ and a large number of retail outlets, for each of these considerations makes reduction in safety allowances possible. The level of carrying charges and of the costs of depletion also are of extreme importance in determining the proper method of

[17] In the case of certainty, this function is not needed.

distribution. Increases in the cost of depletion bring about a lowering of the optimal probability of depletion, leading to increased safety allowances and thus to increased probability that central storage is desirable.

Changes in the level of carrying charges present a somewhat more complicated problem. An increase in carrying charges should have two effects, namely, (1) to reduce the level of protection against depletion and (2) to increase the amount of saving per unit of reduced safety allowances. Effect (1) dominates (2) for high probabilities of depletion, while for low probabilities of depletion, (2) dominates (1). Therefore, for high probabilities of depletion, increases in carrying charges decrease the likelihood of central storage being desirable. Conversely, for low probabilities of depletion, increases in carrying charges increase the likelihood of central storage.

Reductions in safety allowance constitute a field where tremendous savings may be possible. An example of the ridiculous level to which safety allowances can rise is described in the Bonner Committee Hearings where it is pointed out that 335 cans of paint in the pipeline from the United States to Germany were required to supply the using unit with one can of paint a day for maintenance purposes.[18] Rational analysis should illuminate the wastes involved in a system that allows such situations.

Several aspects of a realistic model have been omitted intentionally, for the sake of ease of presentation. For example, purchasing costs have not been included, although methods of incorporating them into the analysis have been described elsewhere.[19] Quantity discounts and price anticipations are among the other factors that have been omitted. Nevertheless, the model does indicate that in some situations there is a need for various types of supply, some involving central storage, others not. This aspect of the model in a sense corresponds with reality.

In locational models the conventional method of approach postulates the existence of a central office and then decentralization of functions in some time sequence. Perhaps a study of large mail order companies would shed light on this problem. The main advantage of increasing the number of mail order plants is that better customer service is provided and sales are increased. On the other side of the ledger, inventory levels and transportation costs must be increased. Attempts should be made to quantify these various aspects of the problem, and thus rationally arrive at the decision as to how many outlets are desirable.

The problem of determining how many mess halls to have on a naval station possesses the essential characteristics of our locational model. From the point of view of the amount of food required, the smaller the number of mess halls, the more economical the feeding.[20] On the other hand, if there are few mess halls, the men must come from farther away. Thus time and expense are involved in getting the men to the food. A balancing of these advantages and disadvantages should help to determine the desirable number of mess halls.

The interrelationships between the various models considered here are sometimes of a fairly simple nature. For example, some aspects of the location model can be looked at as changes in the time factors in the safety-allowance model. In the decision model, revised demand estimates fed back from the information center can be used for the safety-allowance model's demand estimates.

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[18] Federal Supply Management Hearings, Feb. 22, 27, 28 and March 5, 11, 1952, p. 42.

[19] T. M. Whitin, The Theory of Inventory Management, Princeton, 1953, pp. 56-62.

[20] See page 29 above. Of course, if this principle is carried too far, diseconomies may set in because of type of equipment.

We now come to the purchase-quantity model. It was mentioned above that the determination of economic purchase quantities leads to economies of scale. More specifically it has been shown by many authors that the best balance between ordering costs and carrying charges may be achieved by ordering a purchase quantity equal to $\sqrt{2YS/I}$, where S represents the cost of placing an order, Y represents annual sales in dollars (at cost prices), and I represents carrying charges (as a percent of cost). Such a formula, however, is based upon the supposition that only one order is required for a purchase. In the event that the "economic purchase quantity" exceeds the capacity of the firm with which the order is placed, the simple formula is no longer applicable, although it can be modified to take this difficulty into account.

The much-discussed Paint Study carried out for the Munitions Board by a Harvard Business School group was an attempt to evaluate the effects of centralization on purchasing. The results of this study often have been cited as indicating that single-department procurement does not lead to savings. The study is inconclusive, however, in that it merely indicated that single-department procurement did not lead to savings in one particular instance. The reasons for this failure may be eliminated. For example, consider the argument that a very large purchase quantity enables producers to charge a high price, for they know that several bidders will receive contracts. Surely, it is possible for the single department to space purchases in at least as efficient a manner as the individual services could. Furthermore, the single-department procurer could exert more powerful monopsonistic influence. One of the reasons that this possibility did not materialize is that the relative urgencies of the orders of the departments was not evaluated. A revised theory of priorities and more standardization of requisitioning procedures should help to eliminate this difficulty.

Concerning the problem of unification of supply, no one is yet in a position to evaluate the merits and demerits of a unified system. The facts are not yet known. Nevertheless, to clarify some of the issues at stake, I should like to make the following remarks:

(1) A commonly expressed argument is (a) that unification of supply systems brings about centralization; (b) that General Motors has found decentralized operations profitable; and therefore (c) unification is bad. As indicated in the first part of this paper, unification does not necessarily involve centralization. Therefore, this argument is clearly invalid. Furthermore, the most compelling reasons usually given for decentralization of large corporations involve the use of the price mechanism as an indicator of the relative profitability of the various units.[21] Since such analysis is not applied by the military services, all the decentralization arguments related to the price mechanism are irrelevant.

(2) From the safety allowance standpoint, economies would be achieved by reducing the number of pipelines. Obvious though this statement may be, a recent statement by an individual in an extremely high position specifically denied that inventories could be reduced in this way because stock levels depend on over-all military requirements.

(3) The amount of material in military pipelines is tremendous, as the following statement before the Bonner Committee indicates:[22]

> Those two pipelines (Army and Navy), at the end of the war in ETO, had about as much material in them as was consumed during the entire World War II. The Army alone reports $31,464,000,000 in supplies on hand at the end of World War II, exclusive of subsistence or petroleum products....

[21] R. A. Dahl and C. E. Lindblom, Politics, Economics, and Welfare, New York, 1953, pp. 378 ff.

[22] Federal Supply Management, Sixth Intermediate Report of the Committee on Expenditures in the Executive Departments, June 27, 1951, p. 79.

The savings involved in reducing these pipelines would therefore be of a large order. For example, it has been stated that:[23]

> Cooperative action at the New York office demonstrated that under a unified operation it is possible to look for tremendous savings from intangible economic factors. These embrace the more efficient utilization of production facilities and materials, less competition for productive capacity, more favorable prices, improved planning and scheduling of supply requirements and better purchasing methods, as well as the reduced overhead of consolidated operations. If Army and Navy procurement of quartermaster common items, exclusive of subsistence, fuels, and lubricants, had been combined during the war, this ratio would have yielded savings amounting to $1,116,000,000.

(4) The Alameda Medical Supply Test indicated that there are possibilities of large savings by unified distribution of medical supplies. Alameda provides a representative cross-section of supply support for all three services in the Western States and in addition, supplies Korean operations.

> At the West Coast site selected for the medical test, the Army previously maintained a medical depot at Oakland, California, with 344,000 square feet of storage space. Across the street, the Navy had a similar depot with 535,000 square feet of space. The Alameda depot with 677,000 square feet of space replaces these two installations totaling 879,000 square feet of space. This represents a saving of some 200,000 square feet of valuable space.[24]

(5) The Report of the President's Committee on Administrative Management stated:[25]

> The conspicuously well-managed administrative units in the government are almost without exception headed by single administrators.

Although such evidence is not conclusive, it should be given serious consideration, particularly when the Bonner Committee reports:[26]

> On the basis of field observations to date and other data available, supply management in the Department of Defense and the component military departments lacks adequate centralized direction and coordination.

On the opposite side of the ledger, one often finds extremely weak arguments, of which the following quotation is typical:[27]

> If the business can be carried on satisfactorily without a new executive level or without a new functional executive, a change in the degree of centralization is neither necessary nor desirable.

(6) At the recent Supply Management Conference it was contended that the superficial thinking that leads to the false conclusion that there is an inherent virtue in centralization and standardization could be rejected. It cannot be denied that there are certain virtues inherent in

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[23] Federal Supply Management, Hearings, Feb. 22, 27, 28 and March 5 and 11, 1952, pp. 23-24.

[24] Alameda Medical Supply Test, Seventeenth Intermediate Report of the Committee on Expenditures in the Executive Departments, Government Printing Office, 1952, pp. 2, 4. At $8 per square foot, this represents a saving of about 1.5 million dollars.

[25] On p. 32.

[26] Federal Supply Management, Sixth Intermediate Report of the Committee on Expenditures in the Executive Departments, June 27, 1951, p. 3.

[27] Emergency Management of the National Economy, U. S. Industrial College of the Armed Forces, Washington, 1950, p. 95.

centralization and standardization. On the other hand, there are many examples of the diseconomies involved. The question is one of balancing these economies and diseconomies. Concerning standardization, it is clear that where standardization can be effected without impairment of technical efficiency it leads to savings through reduced inventory levels.

(7) Concerning responsiveness to command as an argument against unification, the following can be said:

(a) That the various commands operating separate supply systems cannot act in the best interests of the nation, in wartime, for the over-all allocation of the nation's scarce resources must be considered.

(b) The fact that the Air Force has been supplied by the Army without raising a large number of problems of this type is evidence that a unified system may work.

I wish to emphasize in closing that I am not here contending that unification is desirable. Good reasons for the maintenance of separate supply systems may be advanced. It is my contention, however, that the opponents of unification have not yet presented a good case, as the above remarks indicate. The following paragraphs are for the purpose of outlining a few possible approaches that could be but have not been attempted by the proponents of separate supply systems for the various services.

A study of the costs of collecting and processing information might show that these costs rise at an extremely rapid rate with increases in the size of organization. If such is the case, increases in the size of organization might well involve prohibitive costs. One cannot prejudge the results of a study of the information and decision-making aspects of military organizations. It is quite possible, however, that they may bring about important diseconomies of scale.

Finally, a focus on certain aspects of the four models discussed above gives rise to several different sources of diseconomies of scale. In model (1), the decision-making model, a rapid increase in the costs of transmitting and processing information might make increased centralization unprofitable. The value of the time lost in transmitting information might outweigh the value of the information. Furthermore, as mentioned above, if the nature of the interdependence between the various firms is not known, it may be desirable for each firm or supply center to make its own decisions.

In model (2), the safety-allowance model, it may be argued that the demand distributions are not known and that the individuals estimating demand are unwilling and unable to conceive of demand as a probability distribution. It may be difficult or impossible to estimate the relevant biases. Furthermore, the random element may be of negligible importance in the overall picture. Also, since the problem of "strategic value" has not been solved, it is currently impossible to set optimal safety allowances for items of military equipment.

Locational models such as model (3) are different in the case of items of military supply than for other items because of vulnerability to bombing. Thus, it might be argued that duplication of pipelines is not a waste, for it may lessen the probability of damage through enemy action. If damage to the supply lines of one service does not involve damage to those of the other services, there will be less confusion resulting therefrom.

In model (4), the purchase-quantity model, it is possible that more information about purchasing plans may leak out under a unified supply system. This leakage might enable entrepreneurs to collude and to charge higher prices than would otherwise be the case.

These arguments are not conclusive without adequate data to back them up. Nevertheless, they are sounder logically than the typical arguments against unification discussed

above.[28] It is hoped that further research will clarify the issues at stake and pave the way for an eventual decision based on the true merits of the case rather than on personal biases. It is likely that the use of formal models will contribute in this direction.

[28]On pp. 32-33.

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# THE DUAL METHOD OF SOLVING THE LINEAR PROGRAMMING PROBLEM *

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Many problems in logistics are capable of being formulated as linear programming problems. For example, one such problem is the so-called transportation problem, which is concerned with obtaining the best way (in the sense of least time, or of least expense, etc.) of shipping stipulated quantities of materials from m origins to n destinations. This paper develops a new computational method of solving general linear programming problems.

This paper presents a method of solution of linear programming problems which developed from considerations brought to the fore in "A Modified Simplex Method for Control of Round-off Error in Linear Programming" [1], namely the exploitation of orthogonality and the dual theorem of Tucker, Kuhn and Gale [2]. The resulting method, which I call the "dual" method (polar dual might be more accurate since there is a change from point to hyperplane as the fundamental geometric object), (1) yields the same sort of computational advantages as the "modified simplex" method, (2) eliminates the need for doubling the number of variables where they are not necessarily nonnegative, (3) focuses on a simultaneous geometry of the direct and dual problems which may be advantageous in particular problems, which provides a visualization of the direct and dual problems at every stage of the simplex procedure, and which offers the eventual possibility of a combination of the two methods to obtain solutions efficiently in "pincers" fashion. Incidentally, proofs of all propositions stated without proof in the "modified simplex" paper are supplied in the course of development of the new method.

We shall first pose the "simplex" problem (I name it this in honor of G. B. Dantzig's method of solution) and the dual problem, and relate them by the fundamental dual theorem of Tucker, Kuhn, and Gale.

Given m-dimensional vectors (real m-tuples) $P_0, P_1, \ldots, P_n$ and n real scalars, $c_1, c_2, \ldots, c_n$, the two problems are:

I. The simplex problem:

Minimize

$$Z_0 = \sum_{j=1}^{n} \rho_j c_j, \tag{1}$$

where

$$P_0 = \sum_{j=1}^{n} \rho_j P_j, \text{ and } \rho_j \geq 0,\ j = 1, 2, \ldots, n. \tag{2}$$

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II. The dual problem:

Maximize

(3) $P_0 w$,

where

(4) $P_j w \leq c_j$, $j = 1, 2, \ldots, n$.

($P_k w$ is the inner product of $P_k$ and w).

THE DUAL THEOREM: If either min $Z_0$ of problem I or max $P_0 w$ of problem II has a finite value, then so does the other and:

(5) $\min Z_0 = \max P_0 w$.

We proceed now to a discussion of the dual method of solving the dual problem. We assume that $\max P_0 w < \infty$, and that one may find m linearly independent vectors among $P_1, P_2, \ldots, P_n$. Also, for simplicity of exposition, we shall initially assume that if $w_0$ is the (unique) point satisfying

(6) $P_{r_i} w_0 = c_{r_i}$, $i = 1, 2, \ldots, m$,

where $P_{r_1}, P_{r_2}, \ldots, P_{r_m}$ are linearly independent, and $P_j$ is not one of the $P_{r_i}$, then

(7) $P_j w_0 \neq c_j$.

Later, in Appendix I, we show how we may remove this restriction.

Suppose we have located a point $w_0$ satisfying the restrictions (4) and also satisfying (6) for some linearly independent vectors $P_{r_i}$. We call such a point an extreme point solution (e.p.s.) for the dual problem. It is convenient to relabel the $P_{r_i}$ as $a_i$, $i = 1, 2, \ldots, m$. Since the $a_i$ form a basis for $V_m$, the collection of all m-tuples, we may locate another (unique) basis, $a^1, \ldots, a^m$, of vectors satisfying the relations

(8) $a_i a^j = \delta_{ij}$, $i,j = 1, \ldots, m$,

where $\delta_{ij} = 0$ for $i \neq j$; $\delta_{ii} = 1$ for $i = j$. (If A is the m×m matrix whose rows are the $a_i$, then the columns of $A^{-1}$ are the $a^j$.) The employment of the vectors $a^i$ will simplify our manipulations considerably. If x and y are any m-tuples we may write

(9) $$x = \sum_{i=1}^{m} \theta_i a_i,$$

exhibiting x as the uniquely determined linear combination of the $a_i$, and solve for the $\theta_i$ via (8) by

(10) $$a^j x = \sum_{i=1}^{m} \theta_i (a_i a^j) = \theta_j,$$

so that

(11) $$x = \sum_{i=1}^{m} (x a^i) a_i = \sum_{i=1}^{m} (x a_i) a^i,$$

and

(12) $$xy = \sum_{i=1}^{m} (x a^i)(y a_i).$$

In particular, we have

(13) $$P_0 = \sum_{i=1}^{m} (P_0 a^i) a_i,$$

(14) $$P_j = \sum_{i=1}^{m} (P_j a^i) a_i.$$

Now having an e.p.s. $w_0$, we devise an optimality criterion which allows us to decide whether or not $P_0w_0$ yields the maximal value, and, if not, how we may proceed to another e.p.s., at the same time increasing the value of the functional (3). We then show that this process must end in a finite number of steps, so that some e.p.s. yields the maximal value. Such an e.p.s. we call a maximal e.p.s. (m.e.p.s.).

The criterion is the following:

One checks the signs of the quantities $P_0a^i$ Either

A. $P_0a^i \geq 0$ for all i, or

B. $P_0a^i < 0$ for some i.

Suppose that A holds. Then (13) is a solution to the simplex problem, whose corresponding functional value is

$$Z_0 = \sum_{i=1}^{m} (P_0a^i)c_{r_i} = \sum_{i=1}^{m} (P_0a^i)(w_0a^i) = P_0w_0. \tag{15}$$

The dual theorem then states that $P_0w_0$ is the common optimal value for the two problems, and we are finished.

If B holds (i.e., for i = s, say), we have $P_0a^s < 0$. Let

$$\overline{w} = w_0 - \theta a^s. \tag{16}$$

Now for any value of $\theta$ we retain

$$a_i\overline{w} = a_iw_0 - \theta a_ia^s = a_iw_0 = c_{r_i}, \text{ for } i \neq s, \text{ whereas for } \theta > 0 \text{ we have} \tag{17}$$

$$a_s\overline{w} = a_sw^0 - \theta a_sa^s = c_{r_s} - \theta < c_{r_s}. \tag{18}$$

For any $P_j$ not an $a_i$ we have

$$P_jw_0 < c_j, \tag{19}$$

by assumption. If, for any such $P_j$ we have $P_ja^s \geq 0$, then for any positive $\theta$ we have

$$P_j\overline{w} = P_jw_0 - \theta P_ja^s \leq P_jw_0 < c_j. \tag{20}$$

If $P_ja^s \geq 0$ for all $P_j$, then $\overline{w}$ would be a solution to (4) for any positive $\theta$ and

$$P_0\overline{w} = P_0w_0 - \theta P_0a^s \tag{21}$$

could be made arbitrarily large so that max $P_0w = \infty$, contrary to assumption. Hence, for some j, $P_ja^s < 0$ and choosing

$$\theta = \min (P_jw_0 - c_j)/P_ja^s \text{ for } P_ja^s < 0, \tag{22}$$

we obtain a positive $\theta$. Then $\overline{w}$ satisfies (4) and the value $P_0\overline{w}$ given by (21) where $\theta$ has the value (22) is greater than $P_0w_0$. Now the value (22) of $\theta$ is taken on for j = k, say, so that

$$\theta = (P_kw_0 - c_k)/P_ka^s. \tag{23}$$

Then $\overline{w}$ satisfies the m-1 equalities (17), as well as

$$P_k\overline{w} = P_kw_0 - \theta P_ka^s = P_kw_0 - (P_kw_0 - c_k) = c_k. \tag{24}$$

It is a simple matter to show (Section 2 of [3]) that the vectors $a_1, \ldots, a_{s-1}, P_k, a_{s+1}, \ldots, a_m$, are again linearly independent, so that $\overline{w}$ is another e.p.s. Finally, the value (22) is taken on for only $P_k$, since if $P_{k'}$ were to yield $\theta$ also, we would have $P_{k'}\overline{w} = c_{k'}$, contrary to the assumption described by equation (7).

We may now express $P_0$ and $P_j$ in terms of the new basis and reapply the criterion.

Now B cannot continue to hold indefinitely, since there are only a finite number of possible e.p.s.'s, and since any e.p.s. appears at most once, because of the increase in the functional at each stage. Hence A must be attained in a finite number of stages.

This completes the description of the step-by-step procedure in the dual method, aside from the modifications necessary when the restrictive assumptions made above are relaxed. We discuss the relaxation in Appendices I and II which contain, respectively, the treatment of "degeneracy" and the treatment of obtaining an initial e.p.s.

To relate geometrically the simplex problem as solved by the simplex method and the dual problem treated by our method, we shall next state the requisite facts concerning the simplex method.

Suppose one has a solution (2) to problem I for which the vector coefficients of the positive $\rho_j$ are linearly independent. Such a solution we shall call a basic solution (b.s.). Let the number of linearly independent vector coefficients of the positive $\rho_k$ be $m' \leq m$. If $m' < m$ we say we are at a degenerate stage, or that degeneracy has occurred. This situation poses special difficulties. However, A. Charnes [4] has shown how we may alter the problem initially, obtaining an entirely equivalent problem, in such a manner that degeneracy never occurs. The essential effect of such an alteration is that we may initially suppose that $P_0$ is independent of any m-1 vectors selected from among $P_1, \ldots, P_n$. We also suppose, as before, that $\min Z_0 > -\infty$ and that we may find a basis from the vectors $P_1, \ldots, P_n$.

Under these assumptions, Dantzig [5] has established an optimality criterion whereby one may decide whether or not a current b.s. is a minimal solution (m.b.s.) and, if not, how one may obtain another b.s. which yields a smaller value of $Z_0$. He then shows that a finite number of stages is sufficient to yield the minimum.

We now establish the optimality criterion in our present notation since we require the formulas for the geometric interpretation.

If we have a b.s., we shall call the linearly independent vector coefficients of the positive $\rho_j$, $a_1, a_2, \ldots, a_m$, and, as before, express the vectors $P_0, P_1, \ldots, P_n$ in terms of the $a_i$ as in (13) and (14). Again, let $w_0$ be the unique point satisfying (6).

The criterion is the following:

One checks the signs of the quantities $P_j w_0 - c_j$. Either

A. $P_j w_0 - c_j \leq 0$ for all j, or

B. $P_j w_0 - c_j > 0$ for some j.

Suppose that A holds. Then $w_0$ satisfies (4), constituting a solution to the dual problem whose corresponding functional value is

$$P_0 w_0 = \sum_{i=1}^{m} (P_0 a^i)(w_0 a_i) = \sum_{i=1}^{m} (P_0 a^i) c_{r_i} = Z_0. \tag{25}$$

Then again the dual theorem states that $P_0 w_0$ is the common optimal value for the two problems.

If B holds, say for $j = k$, we have $P_k w_0 - c_k > 0$, and we write

$$P_0 = \sum_{i=1}^{m} (P_0 a^i) a_i - \theta P_k + \theta P_k = \sum_{i=1}^{m} (P_0 a^i - \theta P_k a^i) a_i + \theta P_k. \tag{26}$$

Now if it could be that the $P_k a^i$ were non-positive for all i, then (26) is a solution to problem I for any positive $\theta$, so that taking $\theta$ arbitrarily large, the value of the functional

$$Z_0 = \sum_{i=1}^{m} (P_0 a^i - \theta P_k a^i) c_{r_i} + \theta c_k = \sum_{i=1}^{m} (P_0 a^i) c_{r_i} - \theta (P_k w_0 - c_k) \tag{27}$$

could be made arbitrarily small; i.e., min $Z_0 = -\infty$, contrary to assumption. Hence, for some i, we have $P_k a^i > 0$, and choosing

$$\theta = \min P_0 a^i / P_k a^i, \text{ for } P_k a^i > 0, \tag{28}$$

then (26) for $\theta$ given in (28) is a solution to the problem which yields a smaller value of the functional given by (27) with $\theta$ as in (28). We show that (26) is again a b.s. In fact, for some value of i, say i = s, we have

$$\theta = P_0 a^s / P_k a^s, \tag{29}$$

and the coefficient of $a_s$ in (26) is 0. As before, the vectors $a_1, \ldots, a_{s-1}, P_k, a_{s+1}, \ldots, a_m$ form a basis, so that (26) is indeed a b.s. The nondegeneracy assumption again asserts that all the coefficients of the vectors in (26), with the exception of $a_s$, are positive.

Having the new b.s. we may reapply the criterion. As in the dual method, we conclude that A must eventually be true and that in a finite number of stages one attains an m.b.s.

We may now develop the simultaneous geometrical representation to which allusion has been made.

Consider, in the vector space $V_m$ of all m-tuples, the set $\Lambda$ of all points w satisfying the n relations (4). The set $\Lambda$ is a convex set. That is, if $w_1$ and $w_2$ are points of $\Lambda$, then, for $0 \le \theta \le 1$, the point $\theta w_1 + (1-\theta) w_2$ is also a point of $\Lambda$, since

$$P_j[\theta w_1 + (1-\theta) w_2] = \theta P_j w_1 + (1-\theta) P_j w_2 \le \theta c_j + (1-\theta) c_j = c_j. \tag{30}$$

DEFINITION: A point p of a convex set K is called an extreme point (e.p.) of K if p cannot be written as 1/2 the sum of any two distinct points of K.

Now the set $\Lambda$ defined above is also a convex polyhedron in the sense that it consists of all points common to a finite number of half-spaces, namely the n half-spaces (4). In the case of a convex polyhedron $\Lambda$, the extreme points may be characterized as follows: [3]

$w_0$ is an e.p. of $\Lambda$ if

(i) $w_0$ is a point of $\Lambda$

(ii) there exists m linearly independent vectors $P_{r_i}$, chosen from among the $P_j$ and satisfying (6).

These e.p.'s include the e.p.s. we utilized in the dual method. We say that two such e.p.'s are adjacent if they are distinct and jointly satisfy m-1 of the relations (6). Consider two adjacent e.p.'s. The line segment joining them has the property that any point on it also satisfies the m-1 relations from (6) common to the two e.p.'s. We may call such a line segment a finite edge of $\Lambda$.

We recall that in the dual method, associated with each e.p.s., $w_0$ not an m.e.p.s., was another e.p.s., $\overline{w} = w_0 - \theta a^s$, and that $w_0$ and $\overline{w}$ satisfied jointly all of the relations (6) except $a_s w = c_{r_s}$. That is, $w_0$ and $\overline{w}$ were adjacent e.p.'s, and the line segment joining them is the set of points $w_0 - \phi a^s$, for $0 \le \phi \le \theta$, and $a^s$ is parallel to this line segment.

We may thus describe the dual method as one which proceeds from e.p. to adjacent e.p. of $\Lambda$ until a maximal e.p. is found.

Recall next that in the simplex method, associated with each b.s., was a point $w_0$ satisfying property (ii) of the preceding characterization of an e.p. of $\Lambda$. We call such a point $w_0$ a base point (b.p.) for the simplex problem. Then the two b.p.'s associated with two consecutive steps in the simplex method also have m-1 of the relations (6) in common. Two such b.p.'s will be called adjacent b.p.'s. Then the simplex method may be described as a method of proceeding from b.p. to adjacent b.p. until a minimal b.p. (m.b.p.) is located. The criterion then asserts, via the dual theorem, that such an m.b.p. is also an m.e.p. of $\Lambda$.

Now consider the entire set of points $w_0$ which satisfy m linearly independent equations from among the n equations:

(31) $P_j w = c_j$, $j = 1, \ldots, n$.

The total possible number of such points is $n!/(n-m)!m!$, or the total number of different ways one can take m things from n things. We shall call these points critical points (c.p.). As we have noted, when a c.p. is in $\Lambda$ it is an e.p. of $\Lambda$, whereas, if $P_0$ may be expressed as a positive combination of the basis vectors associated with a given c.p., as in (13), then that c.p. is a b.p. for the simplex problem. Of course, some c.p.'s may not fall into either category.

Now consider the set of points w satisfying

(32) $P_0 w = k$.

The set of all such points w determine what is called a hyperplane of $V_m$. As k varies, in particular as k increases, we visualize the variable hyperplane (32) moving through $V_m$ and through $\Lambda$, parallel to itself in the direction of the positive normal $P_0$.

If M is the maximum of the functional $P_0 w$ as w ranges over $\Lambda$, we visualize the optimal hyperplane

(33) $P_0 w = M$

"resting" on the set $\Lambda$, meaning that the entire set $\Lambda$ lies on the nonpositive side of (33), since if $w_1$ is in $\Lambda$, then $P_0 w_1 \leq M$, whereas $\Lambda$ meets (33) in at least one point. Now consider any b.p. $w_0$ for the simplex method. The dual theorem asserts that $P_0 w_0 \geq M$; the equal sign holding only if the b.p. is an m.b.p. That is, every b.p. lies on the nonnegative side of (33).

All of the preceding points of discussion are clearly visualized in a $V_2$ or 2-plane. The "hyperplanes" (31) and (33) then become merely lines, with positive normals $P_j$ and $P_0$, respectively.

Referring to Diagram 1 where m = 2 and n = 6, the normals $P_1, \ldots, P_6$ of the six hyperplanes $P_j w = c_j$ have been drawn, and the convex set $\Lambda$ is seen to be the set of all points common to the nonpositive sides of these hyperplanes. In the simplex method, possible b.p.'s are $w_0^1$, $w_0^2$, F, G, and a minimal b.p. is $\overline{w}_0$. Thus, a possible procedure in the simplex method would be from $w_0^1$ to $w_0^2$ and thence to $\overline{w}_0$ (not from $w_0^1$ directly to $\overline{w}_0$ in one stage). Other possible simplex procedures would be from $w_0^1$ to G to $\overline{w}_0$ or from F to $\overline{w}_0$ (we need not stop at G). Note that at the point $w_0^1$ we have drawn the two normals, $a_1 = P_3$ and $a_2 = P_5$, indicating that $P_0$ is expressed as a positive combination of the basis vectors $a_1$ and $a_2$. Thus, (13) would read

(34) $P_0 = (P_0 a^1) a_1 + (P_0 a^2) a_2$,

where $P_0 a^i > 0$, $i = 1, 2$. This has also been indicated at the points $w_0^2$ and $\overline{w}_0$.

The optimal "functional" hyperplane $P_0 w = M$ is shown "resting" on $\Lambda$. $P_0$ has been drawn vertically arbitrarily.

The extreme points of $\Lambda$ are A, B, C, D, E, and $\overline{w}_0$, each lying on m = 2 independent hyperplanes. Adjacent extreme points are, for example, A and E, or C and D. If in the dual method one were at A, for example, one might proceed from there to E and thence to $\overline{w}_0$, or from A to B to C to D to $\overline{w}_0$ (not from A to C, for example). Critical points which are neither e.p.'s nor b.p.'s are H, G, K, and L.

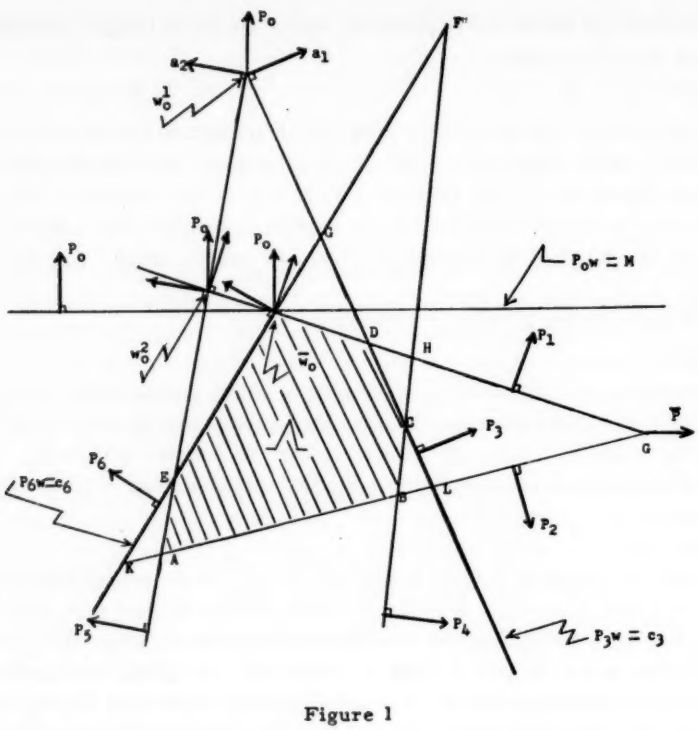


Figure 1

## APPENDIX I

## DISCUSSION OF THE CASE OF DUAL DEGENERACY

Recall that, in the dual method, in moving from e.p.s. to e.p.s. we made the assumption that when $w_o$ was an e.p.s. satisfying

(I.1) $\quad a_i w_o = c_{r_i}, \; i = 1, \ldots, m,$

and when $P_j$ was <u>not</u> an $a_i$, we had

(I.2) $\quad P_j w_o < c_j.$

Geometrically speaking, the assumption was that an e.p. such as $w_o$ lay on <u>precisely</u> m of the hyperplanes

(I.3) $\quad P_j w = c_j, \; j = 1, \ldots, n.$

One can visualize a possibility of "cycling" when this restriction is lifted. Thus, suppose $w_o$ also satisfied

(I.4) $\quad P_1 w_o = c_1,$

say where $P_1$ is not an $a_i$, and $P_1 a^s < 0$. Recall that the increase in $P_o w$ that was obtained was of the amount $-\theta P_o a^s$ where

(I.5) $$\theta = \min_j (P_j w_0 - c_j)/P_j a^s, \text{ for } P_j a^s < 0.$$

Our criterion would then force us to choose

(I.6) $$\theta = (P_1 w_0 - c_1)/P_1 a^s = 0,$$

thus replacing $a_s$ by $P_1$ to form a new basis, with no increase in the functional $P_0 w$. Geometrically, we are still at the point $w_0 - \theta a^s = w_0$, having merely chosen a different basis from among the $m+1$ vectors $a_1, \ldots, a_m, P_1$ associated with $w_0$. Proceeding in this manner, it is conceivable that one would eventually return to the same basis $a_1, \ldots, a_m$, while in the process retaining the same value $P_0 w_0$ of the functional. No doubt examples illustrating this cycling could be constructed, as has been done in the simplex case.

We shall say that when we have an e.p. lying on more than m hyperplanes, then we have a dual-degenerate problem or dual-degeneracy has occurred.

We propose to resolve this difficulty in the general case. More precisely, we shall (1) pose an equivalent problem in which degeneracy can never occur and whose solution yields the solution to the original, and (2) devise a scheme whereby the usual computations involved are performed only in the original problem, so that if degeneracy does occur the devised scheme would lead the way out of it.

Consider the set $\Lambda$ of all solutions to

(I.7) $$P_j w \le c_j, \quad j = 1, \ldots, n.$$

Consider, for $\epsilon > 0$, the set $\Lambda(\epsilon)$ of all points satisfying

(I.8) $$P_j w \le c_j + \epsilon^j, \quad j = 1, \ldots, n.$$

Obviously $\Lambda(\epsilon)$ contains $\Lambda(0) = \Lambda$. Geometrically, $\Lambda(\epsilon)$ is obtained from $\Lambda$ by "pushing out" each of its "faces" (the hyperplanes (I.3)), in the direction of the positive normal $P_j$, a distance $\epsilon^j/(P_j P_j)^{1/2}$.

We show that one can find $\epsilon_0 > 0$ such that for $0 < \epsilon \le \epsilon_0$ the extreme points of $\Lambda(\epsilon)$ lie on precisely m of the hyperplanes

(I.9) $$P_j w = c_j + \epsilon^j, \quad j = 1, \ldots, n.$$

Thus, suppose, for example, that $a_1, \ldots, a_m$ form a basis chosen from among the $P_j$, and that $w_0(\epsilon)$ satisfies

(I.10) $$a_i w_0(\epsilon) = c_{r_i} + \epsilon^{r_i}, \quad i = 1, \ldots, m.$$

If $P_j$ is not an $a_i$, we have $P_j = \sum_{i=1}^{m} (P_j a^i) a_i$, so that

(I.11) $$P_j w_0(\epsilon) = \sum_{i=1}^{m} (P_j a^i)(a_i w_0(\epsilon)) = \sum_{i=1}^{m} (P_j a^i)[c_{r_i} + \epsilon^{r_i}],$$

which can equal $c_j + \epsilon^j$ for at most n values of $\epsilon$. If we do this for all such $P_j$'s which are not $a_i$'s and for all such bases $a_i$, we see that there can be only a finite number of $\epsilon > 0$ for which an extreme point of $\Lambda(\epsilon)$ lies on more than m of the hyperplanes (I.9). Hence, we may take $\epsilon_0$ to be any positive number smaller than the smallest possible such $\epsilon$. Thus, for $0 < \epsilon \le \epsilon_0$, maximizing $P_0 w$ subject to the restrictions (I.8), we have a non-dual-degenerate problem, and if $w_0(\epsilon)$ is an m.e.p. to this problem, $w_0 = w_0(0)$ solves the original one.

It remains to show that we may always work with the original problem, so that no $\epsilon$ need be written down.

For purposes of computation, when we have an e.p.s. $w_0$, and an associated basis, $a_i$, we record the coefficients of $P_0$ and $P_j$ in terms of this basis together with the quantities $P_j w_0 - c_j$ in a convenient tableau as follows.

Computational Tableau

| $c_j \rightarrow$ | | | $c_1$ | . . . | $c_k$ | . . . | $c_n$ |
|---|---|---|---|---|---|---|---|
| $\downarrow$ | $a_i$ | $P_0$ | $P_1$ | . . . | $P_k$ | . . . | $P_n$ |
| $cr_1$ | $a_1$ | $P_0 a^1$ | $P_1 a^1$ | . . . | $P_k a^1$ | . . . | $P_n a^1$ |
| . | . | . | . | | . | | . |
| . | . | . | . | | . | | . |
| . | . | . | . | | . | | . |
| $cr_s$ | $a_s$ | $P_0 a^s$ | $P_1 a^s$ | . . . | $P_k a^s$ | . . . | $P_n a^s$ |
| . | . | . | . | | . | | . |
| . | . | . | . | | . | | . |
| . | . | . | . | | . | | . |
| $cr_m$ | $a_m$ | $P_0 a^m$ | $P_1 a^m$ | . . . | $P_k a^m$ | . . . | $P_n a^m$ |
| $P_j w_0 - c_j \rightarrow$ | | $P_0 w_0$ | $P_1 w_0 - c_1$ | . . . | $P_k w_0 - c_k$ | . . . | $P_n w_0 - c_n$ |

The original problem, at the stage visualized in the tableau, would become a degenerate one when the value (22) for $\theta$ would be obtained for more than one value of j, i.e., if there were no unique "replacing" vector $P_k$. We devise a scheme whereby the replacing vector is uniquely determined and which will ensure us that "cycling" does not occur.

To do this let us revert to the restrictions (I.8) and suppose that $w_0(\epsilon)$ satisfies (I.10). We may express $w_0(\epsilon)$ in terms of $w_0 = w_0(0)$ by

$$w_0(\epsilon) = w_0 + \sum_{i=1}^{m} \epsilon^{r_i} a_i, \tag{I.12}$$

as is easily verified. Now, for $0 < \epsilon \leq \epsilon_0$ degeneracy cannot occur. Hence, by the text, if $w_0(\epsilon)$ is not an m.e.p.s., we have $P_0 a^s < 0$ for some s and we may move to a new e.p.s. given by

$$\overline{w}(\epsilon) = w_0(\epsilon) - \theta(\epsilon) a^s, \tag{I.13}$$

where

$$\theta(\epsilon) = [P_k w_0(\epsilon) - c_k - \epsilon^k]/P_k a^s, \tag{I.14}$$

for a unique value of k such that $P_k a^s < 0$. Or, using (I.12)

$$\theta(\epsilon) = [P_k w_0 - c_k]/P_k a^s + [\sum_{i=1}^{m} (P_k a^i)\epsilon^{r_i} - \epsilon^k]/P_k a^s. \tag{I.15}$$

Now degeneracy at the stage in question would occur when for at least one other value of j, say k', we had:

$$[P_{k'} w_0 - c_{k'}]/P_{k'} a^s = [P_k w_0 - c_k]/P_k a^s. \tag{I.16}$$

Corresponding to this subscript k' we have a value $\theta'(\epsilon)$ given by

(I.17) $$\theta'(\epsilon) = [P_{k'}w_0 - c_{k'}]/P_{k'}a^s + [\sum_{i=1}^{m} (P_{k'}a^i)\epsilon^{r_i} - \epsilon^{k'}]/P_{k'}a^s.$$

Now if the corresponding first terms in (I.15) and (I.17) are equal, we rely on the second terms to yield the replacing vector, choosing that value of j yielding an algebraically smallest such second term. Referring to the tableau, suppose first that the $a_i$ are so ordered that if $i_1 < i_2$ then $r_{i_1} < r_{i_2}$. Next include k and k' in that ordering. The replacing column is then that column which first yields (working downward) an algebraically smallest $P_j a^i/P_j a^s$ (regarding also the position of $(-1)/P_j a^s$ in the jth column). With this scheme, $\epsilon$ need not be written down and degeneracy is resolved.

## APPENDIX II

## FINDING AN INITIAL EXTREME POINT SOLUTION

In the text it was tacitly assumed, in both the dual method and the simplex method, that initially one had an e.p.s. or a b.s. For the simplex method, the problem of finding an initial b.s. has been satisfactorily treated [5]. In the dual method we have, geometrically, the problem of reaching an initial extreme point of the convex polyhedron of solution points. In fact, the general problem of finding any point of a given convex set is at times a trying one. For the problem at hand the following method is suggested.

Suppose we have located any critical point. Thus, for independent vectors $P_{r_1}, \ldots, P_{r_m}$ we have the point $w_0$ satisfying

(II.1) $$P_{r_i}w_0 = c_{r_i};\ i = 1, \ldots, m.$$

The fact that the convex set of solutions to (4) is independent of the functional vector $P_0$ suggests that we introduce a new functional vector $\overline{P}$, which may be any vector expressible as a positive linear combination of the vectors $P_{r_i}$, associated with the critical point $w_0$. Consider then the new simplex problem

Minimize

(II.2) $$\overline{Z} = \sum_{j=1}^{n} \rho_j c_j,$$

where

(II.3) $$\overline{P} = \sum_{j=1}^{n} \rho_j P_j,\ \rho_j \geq 0,\ j = 1, \ldots, n.$$

We then see that we have a b.s. $w_0$ for this problem and minimizing $\overline{Z}$ will lead us to an m.b.s., which will be also an e.p.s. for the original problem.

As an example, let us refer to Figure 1 of the text. Suppose we are at the c.p. G. This means that we have a basis consisting of $a_1 = P_1$, $a_2 = P_2$. We have drawn $\overline{P}$ as a positive combination of $P_1$ and $P_2$, and minimizing $\overline{Z}$ via the simplex method would lead us to the extreme point C.

## APPENDIX III

## CONTROL OF ROUND-OFF ERROR

In the text, reference was made to a paper [1] suggesting a modification of the usual simplex method. The modification consists of using, at each stage, only the inverse matrix of the current basis and the original data. The main advantages of the method are: (1) the accumulation of round-off errors as we proceed stage by stage is confined entirely to the current inverse matrix, (2) there is no increase in computation due to an initially "sparse" matrix (e.g., one with many zeros) filling up with nonzero entries as one proceeds from stage to stage.

We shall review the method suggested above, indicating also the magnitude of the number of calculations per stage.

Suppose we have a basis $a_1, \ldots, a_m$ chosen from among the initial vectors $P_j$. Recall the use made of the vectors $a^i$. It will be noted that nowhere in the text did we actually need to know these vectors. We only made use of their existence. In fact, the usual stage-to-stage procedure does not obtain the (i, j) entry in a current tableau by calculating the inner product $P_j a^i$, but obtains $P_j a^i$ from the (i, j) entry of the preceding tableau by means of a general algorithm, thus preserving any error contained in the preceding tableau. We propose to use the same algorithm, but only to find the inverse to the current basis in terms of the inverse to the preceding basis.

Suppose now that we have computed the vectors $a^i$. For both the dual and the simplex methods we desire also the computed components of $w_o$, the point satisfying the equalities:

$$a_i w_o = c_{r_i}. \tag{III.1}$$

Let $e_i$ be the point whose ith coordinate is 1 and whose remaining coordinates are 0. We may write:

$$\begin{aligned} a^j &= \sum_{i=1}^{m} (a^j e_i) e_i;\ j = 1, \ldots, m \\ w_o &= \sum_{i=1}^{m} (w_o e_i) e_i. \end{aligned} \tag{III.2}$$

The quantities we agree to compute thus far are the coefficients of the $e_i$ in these equations. The quantities $w_o e_i$ may be obtained from the $a^i$ by the formula

$$w_o e_j = \sum_{i=1}^{m} (e_j a^i)(w_o a^i) = \sum_{i=1}^{m} (e_j a^i) c_{r_i};\ j = 1, \ldots, m. \tag{III.3}$$

For both methods we require, in addition, the quantities

$$P_j w_o - c_j;\ j = 1, \ldots, n,\quad P_o a^i;\ i = 1, \ldots, m. \tag{III.4}$$

In the simplex method, having decided upon the replacing vector $P_k$ we need also

$$P_k a^i;\ i = 1, \ldots, m. \tag{III.5}$$

In the dual method, having decided upon the vector $a_s$ to be replaced, we need also

$$P_j a^s;\ j = 1, \ldots, n. \tag{III.6}$$

Finally, to complete the transition to the next stage we require the inverse to the new basis, $a_1, \ldots, a_{s-1}, P_k, a_{s+1}, \ldots, a_m$. To this end, relabel $a_i = \bar{a}_i$, $i \neq s$; $P_k = \bar{a}_s$.

We thus seek vectors $\bar{a}^i$ satisfying

(III.7) $\quad \bar{a}_i \bar{a}^j = \delta_{ij};\ i,j = 1, \ldots, m.$

Again the inverse to the matrix whose ith row is $\bar{a}_i$ is the matrix whose jth column is $\bar{a}^j$. In terms of the $a^i$:

$$\bar{a}^s = a^s / \bar{a}_s a^s$$

(III.8)
$$\bar{a}^i = a^i - \frac{\bar{a}_s a^i}{\bar{a}_s a^s} a^s;\ i \neq s,$$

as is verified by noting that the $\bar{a}^i$ actually do satisfy the relations (III.7). This completes the enumeration of the quantities to be computed in proceeding from stage to stage in either method. We repeat that these computations are done only on the current inverse matrix, namely the $a^i$, and the original data.

Counting the number of computations we see that we require about $2m + n$ scalar products plus $2m$ divisions, $m^2 + n$ subtractions, and $m^2$ multiplications. This amount is, at worst, equal to that required by the current method. Again, the advantages due to use of original data in regard to sparseness and the ability to improve, by standard procedures, the inverse at each stage should be borne in mind.

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* * *

# LINEAR PROGRAMMING IN BID EVALUATION

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This paper presents the difficulties which arise in a particular Government procurement agency in an attempt to determine the allocation of contracts which would result in the lowest over-all cost to the Government. It is shown how the recently developed mathematical discipline of linear programming can be applied to such problems.

The law, the Armed Services Procurement Act of 1947, requires that "all purchases and contracts for supplies and services (for the Armed Services) shall be made by advertising ... except as otherwise specifically authorized." Such "advertising" is for the purpose of inviting bidders to quote prices, and other conditions, as a basis for contracts under which the requirements of the Services will be furnished. These requirements are expressed in terms of specifications covering what is wanted and delivery schedules stating when and where the needed items are to be delivered. The law further specifies that, "All bids shall be publicly opened at the time and place stated in the advertisement. Awards shall be made with reasonable promptness by written notice to that responsible bidder whose bid, conforming to the invitation for bids, will be most advantageous to the Government, price and other factors considered."

At first glance it would seem simple enough to determine the "low bidder" from among a number of potential contractors. But there are complications. Perhaps the most involved situation is to be found in connection with "cut-make-and-trim" procurements under which the Government contracts for the manufacture of clothing. In general, the Government furnishes the necessary textiles and findings to the contractor who undertakes—at a stated bid price—to fabricate the required garments. Transportation to and from the contractor's plant is paid for by the Government, except in rare cases in which the contractor can arrange for movement of the material at a lower cost.

Description of the factors which enter into determining the "true cost to the Government" of any prospective clothing contractor's bid is simplified by considering the various elements of cost individually. A full appreciation of the task of expressing and comparing the many possible combinations of price to determine the "lowest true cost" requires an understanding of the interacting influence of the various factors on each other.

The "true cost to the Government" of any garment is the sum of various costs as follows:

(a) The cost of manufacture (fabricating) per unit which varies according to the price quoted in his bid by the manufacturer. An average of about twenty bidders submit quotations under each invitation to bid. A manufacturer may quote different prices for work to be done at different locations. Often discounts for prompt payment of contractors' invoices are offered by bidders. Generally it is feasible to pay such invoices within twenty days; therefore, any

discount offered based on payment within twenty days or more is deducted in stating the cost of manufacture.

(b) The cost of Government-furnished property (textiles) to be furnished the contractor per unit manufactured, which varies according to the percentage of the standard cutting allowance (square yards of material) which the manufacturer states, in his bid, he will require. This statement is predicated on the allocation of an even spread of sizes in an award to any contractor. It is a requirement for efficient cutting of cloth, so as to reduce wastage of the material that a contractor be awarded, within the total quantity he is to produce, a distribution by sizes reasonably comparable with the percentage distribution by sizes in the total quantity covered by the invitation to bid. Since deliveries to specific destinations do not, in general, have the same distribution by sizes as the total quantity being purchased for all destinations, this factor adds greatly to the complexity of deciding upon quantities to be awarded to the various bidders.

(c) The cost of freight charges to ship the Government-furnished property to the manufacturer's plant which varies according to the freight rate between the point of origin of the Government-furnished property and the place of manufacture quoted in each bid. The required Government-furnished property (GFP) may be shipped from the place of production or from various Government depots where it has been stored. Any given item of GFP may be available—on the average—from as many as four different points, each carrying a different freight rate for transportation to the possible points of manufacture of garments. Cost of freight charges for shipping GFP will vary also according to the weight of the GFP, which in turn is dependent upon the percentage of the standard cutting allowance required by a given manufacturer, which, too, varies for different bidders.

(d) The cost of freight for shipment of the finished item which varies according to the freight rate between the different points of manufacture stated in the various bids and the various destinations—averaging 13—to which the Government desires the finished items delivered for storage and further distribution. In the case of freight rates for shipping GFP and the finished item, the rate may vary according to whether commercial or Government bills of lading are used. Further, the cost of freight from the point of origin of the Government-furnished property to the destinations for the finished articles may vary from the sum of the cost of the shipment of Government-furnished property and the finished article if "in-transit freight privileges" are applicable to the given shipment. Such privileges, in practice, permit applying a lower through freight rate to a shipment, even though the movement of the freight is interrupted enroute for processing.

Since October 1, 1952, the procurement of clothing, footwear, textiles, and related items needed by the Department of Defense has been the responsibility of the Armed Services Textile and Apparel Procurement Agency. The operating element of this Agency, ASTAPA, is located in New York City under command of Brigadier General Robert P. Hollis, USA, Chief of Agency Staff.

Shortly after the establishment of ASTAPA, it became apparent that the manual analysis of bids to determine the one which offered the "true lowest cost" did not always result in selection of the right bid or combination of bids. For, in addition to the fundamental factors affecting the "true cost to the Government" of any unit to be purchased, the problem was further complicated for ASTAPA by the inclusion of provisions in the invitation to bid or in the manufacturer's bids. For instance,

(a) Minimum quantities acceptable to the bidder may be stated in the bid. In placing such a restriction on a bid the potential contractor is stating, in effect, that the price quoted is applicable only if at least the stated quantity is awarded to him.

(b) Maximum quantities acceptable also may be expressed in bids. A bidder's plant, or the unobligated productive capacity of his plant, may be such that he can undertake to manufacture not more than a stated quantity.

(c) Both minimum and maximum restrictions in bids may be expressed in terms of "block" or "hinge" bids. In a "block" bid a bidder states that he will accept award only for quantities for delivery to certain specified destinations, or quotes different prices on different quantities he will undertake to furnish. In "hinge" bidding a manufacturer specifies that his quotation on one bid is to be considered applicable only in the event that he does—or does not—receive award under some other bid.

(d) Manufacturers bid on the premise that they will be able to schedule production over the whole period under which deliveries are scheduled in the invitation to bid. In most cases this covers three or six months. But just as the distribution by sizes to any one destination varies from the percentage size distribution for the total quantity being procured, so the delivery schedule for any one destination may vary greatly in quantity by month from the over-all rate of delivery for the whole procurement.

Not every bid includes all the complicating factors enumerated above, but all contain some of the factors and many contain all of them. In some cases the Government finds it suitable to specify that the procurement will be based on delivery at destination and all materials required are to be furnished by the contractor. Except where this condition pertains, the factors listed above result in many different possible combinations of prices. Theoretically, if all resulting combinations are completely compared, the procurement can be made at the lowest possible cost to the Government. However, in many cases, because there are so many different combinations, it is almost physically impossible to exhaust manually all comparisons within a reasonable period of time.

In practice, not all bids received must be subjected to the comparative cost analysis described here. Some bids may have been "unresponsive" in that they offer a nonspecification item or in other ways do not conform to the conditions of the invitation. Some bidders may not be adjudged "responsible" because they may lack adequate facilities or capital to undertake the work. The shops of certain bidders may not be rated as capable of producing garments of the specified quality. The contracting officer weeds out such bids. But there generally remain a number of "responsive" and "responsible" bidders among whom the award, or awards, will depend on comparative, over-all, "true" cost.

A recent typical procurement by ASTAPA resulted in tens of thousands of possible combinations. The actual labor expended to compute and compare these combinations manually approximated 700 man-hours. Estimates showed, however, that if all combinations had been computed the man-hours required for the calculations would have approximated 4000 hours, or one man working full time for somewhat more than two years. There is a real need for rapid analysis of bids, since dealers often limit the effective period of their bids to option periods of twenty days—or fewer—after the opening date.

The practice has been to develop a trend of prices during evaluation rather than exhausting all comparisons, but this method has resulted in uncertainty. The problem, then, is to arrive at a point where it can be stated that any specific purchase has been awarded at absolutely the lowest cost to the Government.

The primary purpose of developing a mechanical solution to the problem is to obtain a technique whereby invitations for bids are evaluated to determine the absolute lowest cost to the Government, with complete accuracy and without qualification.

As a secondary advantage, it is expected that labor costs, as well as the time lag involved from the time a bid is opened until the final award is made, will be reduced considerably through mechanization of bid evaluation. The time and money savings combined with accuracy of bid evaluations are expected to result in over-all economy for ASTAPA, the Services, and the Government.

In addition, an intangible worthy of consideration is the effect more efficient bid analysis will have on the morale of all parties concerned. Absolute accuracy in bid evaluations will reduce contractor criticism of ASTAPA personnel. Quicker award of contracts will minimize inquiries from the trade and allow the purchasing agents, contracting officers, and other operating and administrative personnel to devote more time to other phases of their work. In turn, vendor relations and confidence in ASTAPA will be improved.

Through cooperation with the Office of Naval Research of the Department of the Navy, the problem, in the form of a statement of an actual procurement previously effected by this Agency, was submitted to the George Washington University Logistics Research Project. In turn, personnel of the GWU Logistics Research Project consulted with the Computation Laboratory of the National Bureau of Standards. There resulted a set of instructions to be used on the SEAC, the Bureau of Standards Eastern Automatic Computer, based on the Simplex method of George Dantzig,[1] modified for the general transportation problem, to solve simple contract awards. This code was further improved and its application extended to more general cases by the NBS Computation Laboratory, with the sponsorship of the Office of the Air Comptroller and the Office of Scientific Research, USAF.

The major mechanization problem still to be resolved concerns monthly delivery schedules to individual destinations and the mathematically equivalent size distribution problem. These problems were not submitted to the Logistics Research Project originally, and the development of a rapid technique for their exact solution is the last major step to solve prior to complete mechanization of the problem solution.

We shall now explain the linear programming approach to bid evaluation. Given an homogeneous commodity, a set of manufacturers willing to produce this item at a stated price per item, and various destinations at which this item is desired (which would involve additional freight charge to deliver the commodity) we have almost a statement of the Transportation problem of Hitchcock and Koopmans.[2] Only slight modification need be made to state a simple bid evaluation problem in terms of the framework of the Transportation problem and then to satisfy depot needs and bidder conditions at a minimum over-all cost.

Consider a problem with N depots and M separate bidders. Each of the M bidders wishes to produce an amount not exceeding $a_i$, $i = 1,2,\ldots,M$; and the needs at the N depots are

[1] Activity Analysis of Production and Allocation, T. C. Koopmans, Editor, Wiley and Sons, New York, 1951. This book is frequently regarded as the basic text for linear programming.

[2] T. C. Koopmans, "Optimum Utilization of the Transportation System," Econometrica, Vol. 17, Supplement, July 1949.

$b_j$, $j = 1,2,\ldots,N$. It will cost an amount $c_{ij}$ to deliver a unit from the i-th bidder to the j-th depot; this amount includes the cost of GFP from the nearest source, the stated price per unit less any discount the manufacturer offers, plus the unit shipping charge from point to point. If $X_{ij}$ denotes the quantity purchased from the i-th manufacturer for shipment to the j-th destination, then our problem is to find a set of nonnegative $X_{ij}$ such that:

(1) $\sum_j X_{ij} \leq a_i$ (for each i)

(2) $\sum_i X_{ij} = b_j$ (for each j)

at the same time minimizing

(3) $\sum_{i,j} c_{ij} X_{ij}$.

Note that $\sum_i a_i \neq \sum_j b_j$. In general, we would expect more to be offered for sale than is required in the total award, i. e., $\sum_i a_i \geq \sum_j b_j$. If we establish a fictitious depot, the n-th let us say, which absorbs the amount $\sum_i a_i - \sum_j b_j = b_n$, assigning zero costs $c_{in}$ for all i, we have made the transition from the general transportation problem to the simple bid evaluation problem. (Note that $n = N + 1$). The Simplex method of George Dantzig, easily adaptable for the computation of linear programming problems on a high-speed computing device, may now be applied to determine the contract awards, $X_{ij}$, a problem involving Mn nonnegative variables satisfying M + n equations.

Why do we refer to this as a simple bid evaluation problem? It is because we have not considered the important questions of conditions on the bids such as hinge bidding, multipriced lots from a single bidder, minimum acceptable quantities, scheduling, and the many other factors which may destroy the simplicity of the above formulation.

Some of these conditions, however, can be handled within the framework of this formulation. Let us forget for the moment the scheduling requirement of the producer and the purchaser and enumerate certain types of complications written into the bids to produce for a given contract. Bidder One wishes to make $a_1$ units, but will be happy to receive an award for any part of $a_1$. On the other hand, Bidder Two bids for $a_2$ units and will not accept a contract for less than $a_2$. Bidder Three quotes increasing prices per unit for each of the sublots ${}_1a_3$, ${}_2a_3$ of the total amount he wants to make, $a_3$. To make matters even more difficult, Bidder Four is asking less for each successive lot of $a_4$ on which he is quoting, less for ${}_2a_4$ than for ${}_1a_4$. Although typical of manufacturer's restrictions encountered every day in the ASTAPA program, these are by no means all.

Obviously, if all bids were stated as One's, there would be essentially no difficulty in making the evaluation. A straightforward Simplex method approach would solve for the $X_{ij}$ satisfying the depot needs; the amounts of each bidder not purchased, $a_i - \sum_j^N X_{ij} = X_{in}$, would show up at the fictitious depot; and, since zero cost was associated with all requirements of this station, the resulting $\sum_{i,j}^{M,n} c_{ij} X_{ij}$ would reflect only the cost of actual shipments.

In Bidder Three, we have much the same situation, with the exception that he must be considered as a pair of separate entities. We now must define the possibilities ${}_1X_{3j}$, ${}_2X_{3j}$,

where $\sum_j {}_1X_{3j} = {}_1a_3$, $\sum_j {}_2X_{3j} = {}_2a_3$. Indeed, if z separate lots each having increasing prices are offered by a bidder, he is looked upon as z separate bidders. An award for any or all lots is satisfactory to him. The computation process assures no award for the higher cost lots unless all of the lower priced lots are in the award.

Now we come to Bidder Two and to our first departure from the method outlined above for handling shipments to the n-th (fictitious) depot. Whereas other bidders will tolerate receiving less than their total bid, Bidder Two refuses to do so. If, after a minimum solution analysis, some amount other than $a_2$ is required to be shipped to the fictitious depot, then our solution has not satisfied the condition of the problem. We are bound to consider whether the next best alternative would be to eliminate him from consideration (recomputing with $a_2 = 0$), or to accept his all-or-nothing condition by the ruse of making his price to the fictitious depot, $c_{2n}$, infinitely high, thus forcing $\sum_j^N X_{2j} = a_2$, $X_{2n} = 0$. The cheaper of these is then the minimum feasible solution. Indeed, this approach will account for any minimum condition, not merely an all-or-nothing requirement. If a bidder with a minimum condition is awarded (when one ignores the minimum) a positive amount less than his minimum, then it can be shown that the smaller of the two costs to the government obtained from awarding him nothing or awarding him his minimum is the least cost solution.

Four specifically states, by lowering his price for successive lots, that he offers a premium for buying the previous one. Had he z lots, the z-th could not be purchased unless the (z-1)st were bought, which, in turn, could not be had unless the (z-2)nd had been bought, etc., until the 1st had been purchased.

It is then necessary to consider his bid as z separate offers of z lots, z-1 lots, etc., down to the most costly first lot. Each time, by using the artificially high cost of shipment to the fictitious depot, all lots but the last are required to be bought, i.e., no shipment accepted at the fictitious depot. Each time the following condition is required for the last lot ($\ell$) alone:

$$\sum_j {}_\ell X_{4j} = a_4 \qquad ({}_\ell X_{4n} \text{ possibly} > 0)$$

It remains only to accept the least costly award (all other bidders being satisfied) in deciding which lots, if any, of this bidder ought to be purchased.

One must bear in mind that if more than one bidder, in accordance with the preceding, requires several runs of the problem with different conditions, then all necessary combinations of the conditions of the various bidders must be considered. It is anticipated that, in practice, the number of such combinations will not be excessive. In any case, however, it is the speed of the computer, combined with the elegance of the method, that makes such an examination technologically feasible.

If we were to consider the delivery schedule as extra constraints to the bid evaluation, we would find ourselves away from the simplified statement which enables us to use the Hitchcock-Koopmans transportation problem as a base from which to operate. We have a linear program (solvable by the general Simplex method) which can now be stated.

We shall add a consideration of time to our previous analysis and we shall require the output of each bidder to be proportional to the phasing requirements of the total bid. Instead of $X_{ij}$, we now speak of nonnegative $X_{ij}^k$, which is the amount bidder i ships to depot j in month k. We are now considering depot periods where formerly were just depots. The requirement for

all depots in month k is $T_k$. If T is the total requirement for all depots, then $P_k = \frac{T_k}{T}$ is the proportion of the total bid in the k-th month. The requirement at the j-th depot in month k is $b_j^k$; $a_i$ is the total amount the i-th bidder wishes to produce. The problem is to minimize

$$\sum_{i,j} c_{ij} \left(\sum_k X_{ij}^k\right) \tag{4}$$

But now our sets of constraints are:

$$\sum_i^M X_{ij}^k = b_j^k \qquad \text{(for each pair, j,k)} \tag{5}$$

$$\sum_{j,k}^{N,r} X_{ij}^k \leq a_i \qquad \text{(for each i)} \tag{6}$$

$$\sum_j^N \frac{X_{ij}^1}{P_1} = \sum_j^N \frac{X_{ij}^2}{P_2} = \ldots = \sum_j^N \frac{X_{ij}^r}{P_r} \qquad \text{(for each i)} \tag{7}$$

assuming r months.

The realization that linear programming techniques can be employed to aid in assigning contracts to the proper sets of bidders, and that high-speed computing techniques are available to reduce the elapsed time between opening bids and their complete evaluation, places the ASTAPA organization in the forefront of those organizations with the problem of procurement in common. It is hoped that as this program continues, development in the areas where work is not yet perfected can be undertaken, so that more of the logistic field may be served with an improved type of operation.

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# SOME BLOTTO GAMES

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In many military "games" each "player" must allocate his forces to different "battlefields," "fronts," or "operations areas" before he has a clear picture of the deployment of the enemy. The following paper illustrates how each player should distribute his forces in certain interdiction games.

In one version[1] of the problem of Colonel Blotto, the colonel and the enemy commander distribute their forces among certain forts without knowing the opposing disposition. The particular problem of Colonel Blotto illustrates a general class of "Blotto" games in which

> Two players (A and B) contending on N independent battlefields (labeled 1, 2, ..., N) must distribute their forces (F and G units, respectively) to the battlefields before knowing the opposing deployment. The payoff (a numerical measure of the gain of A or equivalently of the loss of B) on the $i^{th}$ battlefield is given by a function $P_i(x,y)$, depending only on the battlefield and the opposing forces x and y committed to that battlefield by A and B. The payoff of the game as a whole is the sum of the payoffs on the individual battlefields.

An interesting mathematical problem connected with these games is the determination of their solutions from the payoff functions of the individual battlefields. Instead of studying this in general, the present paper illustrates the possible applicability of Blotto games to problems of logistics and tactics by analyzing a particular problem which may be considered as an especially simple Blotto game.

> Suppose a supply system is to deliver a shipment of material from a rear area to an advanced area by one of N independent routes subject to interdiction by enemy assailants.[2] If the route must be selected in ignorance of the interdiction plans of the enemy and the enemy must station his assailants without knowing the route the shipment will travel, the situation described may be regarded as a Blotto game in which A selects the route (battlefield) for the shipment (A's forces) while B distributes his assailants (B's forces) among the different possible routes (battlefields). In this game, $P_i(1,y)$ represents the gain of A (the loss of B) on the $i^{th}$ route when the shipment travels the $i^{th}$ route and y of B's G assailants interdict this route. The analogous quantity when the shipment travels some route other than the $i^{th}$ one is $P_i(0,y)$.

In one military interpretation of this situation the shipment is a naval convoy and the assailants are submarines. In another the shipment is a truck convoy and the assailants are attack aircraft. In a third the shipment is a bombing strike against an enemy target and the assailants are interceptors.

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[1] J. W. Tukey, "A Problem of Strategy," Econometrica, Vol. 17 (1949), p. 73.

[2] By "independent routes" is meant routes such that a single assailant cannot interdict more than one.

The analysis which follows shows how the payoff functions determine a solution to this special type of Blotto game when these functions satisfy the following assumptions:

1. $P_i(0,y) = 0$ for all possible i and y.
2. $P_i(1,y)$ is a strictly decreasing function of y for all i.
3. Either (a) $P_i(1,y)$ is convex in y for all i
   or (b) $P_i(1,y)$ is concave in y for all i.
4. If Alternative (a) of Assumption 3 holds, y is a continuous variable in the closed interval $[0,G]$ and $P_i(1,0) = \lim_{y \to 0} P_i(1,y)$.

Assumption 1 means that the assailants sent against the routes not traveled by the shipment contribute nothing to the payoff. The significance of Assumption 2 is that the amount A gains from the shipment decreases with the number of assailants that interdict its route. Alternative a of Assumption 3 implies that the effectiveness of the assailants used against the shipping route satisfies a law of diminishing returns. The meaning of Alternative b is that this effectiveness satisfies a law of increasing returns. Assumption 4, which requires infinite divisibility of assailants and the continuity of $P_i(1,y)$ as a function of y, is added only to eliminate details which would obscure the reasoning. Omission of Assumption 4 would not basically change the analysis.

**THEOREM** 1: If a Blotto game, in which $F = 1$ and x is restricted to the values 0 and 1, satisfies Assumptions 1, 2, and 4 and Alternative a of Assumption 3, then the value of the game is the unique number V such that

$$\sum_{i=1}^{N} P_i^{-1}(V) = G$$

where $P_i^{-1}(z)$ is defined as the number y such that $P_i(1,y) = z$ for $P_i(1,G) \leq z \leq P_i(1,0)$, as 0 for $z > P_i(1,0)$, and as $G + P_i(1,G) - z$ for $z < P_i(1,G)$. The pure strategy sending $P_i^{-1}(V)$ assailants to interdict the $i^{th}$ route is optimal for B. An optimal strategy for A is to select the $i^{th}$ route for the shipment with probability $q_i$ where the $q_i$ satisfy

$$\sum_{i=1}^{N} q_i = 1$$

$$q_i = 0 \quad \text{if } P_i^{-1}(V) = 0$$

$$q_i \left[ \frac{dP_i(1,y)}{dy} \right]_{y = P_i^{-1}(V)} = q_j \left[ \frac{dP_j(1,y)}{dy} \right]_{y = P_j^{-1}(V)}$$

if $P_i^{-1}(V) > 0$ and $P_j^{-1}(V) > 0$. (If $P_i(1,y)$ is not differentiable at the point $y = P_i^{-1}(V)$, the expression

$$\left[ \frac{dP_i(1,y)}{dy} \right]_{y = P_i^{-1}(V)}$$

is to be interpreted as either the left-hand derivative of $P_i(1,y)$ at $y = P_i^{-1}(V)$, the right-hand derivative, or any number between these two derivatives.)

The convexity of $P_i(1,y)$ insures that $P_i(1,y)$ is continuous for $0 < y < G$. Continuity at $y = G$ follows because $P_i(1,y)$ is convex and decreasing. Continuity at $y = 0$ is postulated in Assumption 4.

The continuity and strict monotoneity of $P_i(1,y)$ imply that $P_i^{-1}(z)$ is a single-valued, decreasing function of z. For $z < P_i(1,0)$ this monotoneity is strict. Therefore

$$H(z) = \sum_{i=1}^{N} P_i^{-1}(z)$$

is a strictly decreasing function for $z < \operatorname{Max}_i P_i(1,0)$. At $z = \operatorname{Max}_i P_i(1,0)$, $H(z) = 0$. At $z = P_i(1,G)$, $H(z) \geq G$. Therefore there is a unique $V \geq \operatorname{Max}_i P_i(1,G)$ such that

$$\sum_{i=1}^{N} P_i^{-1}(V) = G.$$

When $P_i^{-1}(z)$ assailants interdict the $i^{th}$ route, the definition of $P_i^{-1}(z)$ implies that the gain of A if the shipment travels the $i^{th}$ route is z, provided $P_i(1,G) \leq z \leq P_i(1,0)$, and is less than z if $z > P_i(1,0)$. This, coupled with the fact that $V \geq \operatorname{Max}_i P_i(1,G)$, shows that if B assigns $P_i^{-1}(V)$ assailants to the $i^{th}$ route ($i = 1, \ldots, N$), B is assured that A's gains can be no larger than V no matter what route the shipment travels.

If B sends $y_i$ assailants to the $i^{th}$ route ($i = 1, \ldots, N$), the expected payoff when A sends the shipment along the $i^{th}$ route with probability $q_i$ ($i = 1, \ldots, N$) is

$$\sum_{i=1}^{N} q_i P_i(1,y).$$

The difference between this expected payoff and the corresponding expected payoff when B sends $P_i^{-1}(V)$ assailants to the $i^{th}$ route is

$$\sum_{i=1}^{N} q_i \Delta P_i$$

where $\Delta P_i = P_i(1,y_i) - P_i(1,P_i^{-1}(V))$.

Because $P_i^{-1}(V) = 0$ implies $\Delta P_i \leq 0$,

$$\sum_{i=1}^{N} q_i \Delta P_i \geq \sum_{i \in I} q_i \Delta P_i$$

where I consists of those i for which $P_i^{-1}(V) > 0$. From the convexity of the functions $P_i(1,y)$

$$\sum_{i \in I} q_i \Delta P_i \geq \sum_{i \in I} q_i \Delta y_i \left.\frac{dP_i(1,y)}{dy}\right]_{y = P_i^{-1}(V)}$$

where $\Delta y_i = y_i - P_i^{-1}(V)$. (When $P_i(1,y)$ is not differentiable at $y = P_i^{-1}(V)$,

$$\left.\frac{dP_i(1,y)}{dy}\right]_{y = P_i^{-1}(V)}$$

may be interpreted as the left-hand derivative, the right-hand derivative, or any value between.)

Suppose A selects the $q_i$ so that

$$\sum_{i=1}^{N} q_i = 1$$

$$q_i = 0 \quad \text{for} \quad i \notin I$$

$$q_i \frac{dP_i(1,y)}{dy}\Big]_{y = P_i^{-1}(V)} = q_j \frac{dP_j(1,y)}{dy}\Big]_{y = P_j^{-1}(V)} \qquad \text{for } i, j \in I.$$

This selection of probabilities is possible because Assumption 2 implies that the $[dP_i(1,y)]/(dy)$ are all negative and hence the $q_i$ are all positive. With the special selection of the $q_i$,

$$\sum_{i \in I} q_i \Delta y_i \frac{dP_i(1,y)}{dy} \geqq 0$$

because

$$\sum_{i \in I} \Delta y_i \leqq \sum_{i=1}^{N} \Delta y_i = 0$$

This means that if A selects the $q_i$ in the way described above, the minimum expected payoff results when B sends $P_i^{-1}(V)$ assailants to the $i^{th}$ route ($i = 1, \ldots, N$). This minimum is

$$0 \cdot \sum_{i \notin I} P_i(1,0) + \sum_{i \in I} q_i P_i(1, P_i^{-1}(V)) = \sum_{i \in I} q_i V = V.$$

Since both A and B can insure that the expected payoff is at worst V (each from his own point of view), V is the value of the game and the opposing strategies insuring this value are optimal.

THEOREM 2: If a Blotto game in which $F = 1$ and x is restricted to the values zero and one satisfies Assumptions 1 and 2 and Alternative b of Assumption 3, then the value of the game is the unique number V such that

$$\sum_{i=1}^{N} s_i(V) = 1$$

where $s_i(z)$ is the solution of

$$s_i(z)\, P_i(1,G) + (1 - s_i(z))\, P_i(1,0) = z$$

for $z \leqq P_i(1,0)$ and $s_i(z) = 0$ for $z > P_i(1,0)$.

The mixed strategy which sends all G assailants to the $i^{th}$ route with probability $s_i(V)$ is optimal for B. An optimal strategy for A is to select the $i^{th}$ route for the shipment with probability $q_i$ where the $q_i$ satisfy

$$\sum_{i=1}^{N} q_i = 1$$

$$q_i = 0 \text{ if } s_i(V) = 0$$

$$q_i(P_i(1,G) - P_i(1,0)) = q_j(P_j(1,G) - P_j(1,0))$$

if $s_i(V) > 0$ and $s_j(V) > 0$.

From the definition of $s_i(z)$,

$$H(z) = \sum_{i=1}^{N} s_i(z)$$

is a continuous, strictly decreasing function of z for $z \leq \operatorname{Max}_i P_i(1,0)$. Since $H(z) = 0$ for $z = \operatorname{Max}_i P_i(1,0)$ and $H(z) \geq 1$ for $z = P_i(1,G)$, there is a unique V such that

$$\sum_{i=1}^{N} s_i(V) = 1.$$

If B sends all G assailants to the $i^{th}$ route ($i = 1, \ldots, N$) with probability $s_i(V)$, the expected payoff is V when the shipment travels a route for which $s_i(V) > 0$ and is no greater than V if the traveled route has $s_i(V) = 0$. This shows that B can insure that the expected payoff is no greater than V, no matter what route the shipment travels.

An arbitrary mixed strategy for A selects an assignment, $y_1, \ldots, y_i, \ldots, y_N$, of assailants to the N routes according to some probability measure $m(y_1, \ldots, y_N)$ over the space of all possible assignments. The expected payoff when B uses the strategy with measure m and A sends the shipment along the $i^{th}$ route with probability $q_i$ is

$$E(m, q_1, \ldots, q_N) = \int \left( \sum_{i=1}^{N} q_i P_i(1,y_i) \right) dm(y_1, \ldots, y_N).$$

Since $P_i(1,y)$ is concave,

$$P_i(1,y) \geq \frac{y_i}{G} P_i(1,G) + \left(1 - \frac{y_i}{G}\right) P_i(1,0).$$

Therefore

$$E(m,q_1, \ldots, q_N) \geq \int \left( \sum_{i=1}^{N} q_i \left( \frac{y_i}{G} P_i(1,G) + \left(1 - \frac{y_i}{G}\right) P_i(1,0) \right) \right) dm(y_1, \ldots, y_N)$$

$$= \sum_{i=1}^{N} q_i \left( \frac{\bar{y}_i}{G} P_i(1,G) + \left(1 - \frac{\bar{y}_i}{G}\right) P_i(1,0) \right)$$

$$= \sum_{i=1}^{N} q_i P_i(1,0) + \sum_{i=1}^{N} \frac{q_i \bar{y}_i}{G} (P_i(1,G) - P_i(1,0)) \qquad (1)$$

where $\bar{y}_i = \int y_i dm(y_1, \ldots, y_N)$. Suppose A selects the $q_i$ so that

$$\sum_{i=1}^{N} q_i = 1$$

$$q_i = 0 \quad \text{if} \quad s_i(V) = 0$$

$$q_i(P_i(1,G) - P_i(1,0)) = q_j(P_j(1,G) - P_j(1,0))$$

if $s_i(V) > 0$ and $s_j(V) > 0$. Assumption 2 insures that this selection is possible. With this choice of the $q_i$ the last expression in (1), call it K, depends only on $\sum_{i \in I} \bar{y}_i$ where I consists of those i for which $s_i(V) > 0$. Since $P_i(1,G) - P_i(1,0) < 0$, K decreases as $\sum_{i \in I} \bar{y}_i$ increases. Therefore K has its minimum when $\sum_{i \in I} \bar{y}_i = G$. A particular set of values which give this minimum are $\bar{y}_i = s_i(V)G$.

For these values $$K = \sum_{i \in I} q_i \left( s_i(V)P_i(1,G) + \left(1 - s_i(V)\right) P_i(1,0) \right)$$

$$+ \sum_{i \notin I} 0 \cdot \left( s_i(V)P_i(1,G) + (1 - s_i(V)) P_i(1,0) \right) = \sum_{i \in I} q_i V = V.$$

This means that the special selection of the $q_i$ assures A that the expected payoff is at least V, no matter what mixed strategy B uses.

Since both A and B can insure that the expected payoff is at worst V, V is the value of the game and the strategies insuring this value are optimal for A and B.

The two theorems just proved show in two different types of situation how a game-theoretic solution to logistical problems such as that described at the beginning of this paper might be calculated from data describing the logistical and tactical "payoff."

* * *

# OPTIMAL TWO- AND THREE-STAGE PRODUCTION SCHEDULES WITH SETUP TIMES INCLUDED

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Each of a collection of items are to be produced on two machines (or stages). Each machine can handle only one item at a time and each item must be processed through machine one and then through machine two. The setup time plus work time for each item for each machine is known. A simple decision rule is obtained in this paper for the optimal scheduling of the production so that the total elapsed time is a minimum. A three-machine problem is also discussed and solved for a restricted case.

## TWO-STAGE PRODUCTION SCHEDULE

Let us consider a typical multistage problem formulated in the following terms by R. Bellman:

"There are n items which must go through one production stage or machine and then a second one. There is only one machine for each stage. At most one item can be on a machine at a given time.

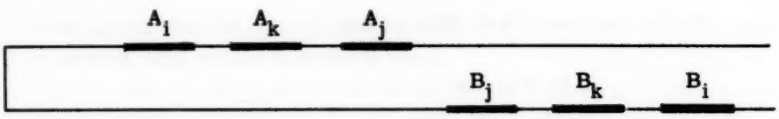
Consider 2n constants $A_i$, $B_i$, $i = 1, 2, \ldots, n$. These are positive but otherwise arbitrary. Let $A_i$ be the setup time plus work time of the i-th item on the first machine, and $B_i$ the corresponding time on the second machine. We seek the optimal scheduling of items in order to minimize the total elapsed time."

A simple decision rule leads to an optimal scheduling of the items minimizing the total elapsed time for the entire operation. For example, the decision rule permits one to optimally arrange twenty production items in about five minutes by visual inspection.

In the second section a three-stage problem is also discussed and solved for a restricted case.

LEMMA 1: The production sequence on either machine can be made the same as that of the other machine without loss of time.

PROOF: On the time scales for each machine place the A's and B's in any position subject to the rules, i.e., the start of a $B_j$ must be to the right of the end of an $A_j$. If the orders are different, the elements out of order will be placed something like the following.

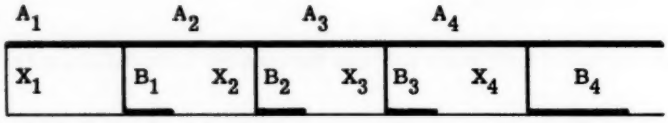



Then without loss of time we can make the ordering of stage 1 the same as the ordering of stage 2 by successive interchanges, starting from the left of consecutive pairs of those items which are out of order. Symmetrically we could order items on stage 2 to match that of stage 1.

Next, since the orders are now the same, we may start each item as soon as possible to minimize the total time. Thus there are no delay times on the first stage.

NOTATION: Let $X_i$ be the inactive period of time for the second machine immediately before the i-th item comes onto the second machine.

If, for example, we consider the sequence S = 1, 2, 3, . . . , n, we have the following time scales for each machine:




We have

$$X_1 = A_1$$

$$X_2 = \max (A_1 + A_2 - B_1 - X_1, 0)$$

$$X_1 + X_2 = \max (A_1 + A_2 - B_1, A_1)$$

$$X_3 = \max \left(\sum_1^3 A_i - \sum_1^2 B_i - \sum_1^2 X_i, 0\right)$$

$$\sum_1^3 X_i = \max \left(\sum_1^3 A_i - \sum_1^2 B_i, \sum_1^2 X_i\right)$$

$$= \max \left(\sum_1^3 A_i - \sum_1^2 B_i, \sum_1^2 A_i - B_1, A_1\right).$$

In general,

$$\sum_1^n X_i = \max_{1 \le u \le n} K_u$$

where

$$K_u = \sum_{i=1}^{u} A_i - \sum_{i=1}^{u-1} B_i .$$

Let

$$F(S) = \max_{1 \le u \le n} K_u .$$

We want a sequence $S^*$ such that $F(S^*) \le F(S_o)$ for any $S_o$.

SOLUTION OF PROBLEM: Consider $S'$ the sequence formed by interchanging the j-th and j+1-st items in S. Then

$$F(S') = \max_{1 \le u \le n} K'_u$$

where

$$K'_u = \sum_{i=1}^{u} A'_i - \sum_{i=1}^{u} B'_i \; ; \; A'_i = A_i \; ; \; B'_i = B_i \quad \text{for } i \neq j, \, j+1,$$

and

$$A'_j = A_{j+1} \; , \; B'_j = B_{j+1} \; , \; A'_{j+1} = A_j \; , \; B'_{j+1} = B_j \; .$$

Then

$$K'_u = K_u \quad \text{if} \quad u \neq j \; , \; j+1.$$

Thus $F(S') = F(S)$ unless possibly if $\max (K_j \; , \; K_{j+1}) \neq \max (K'_j \; , \; K'_{j+1})$.

THEOREM 1: An optimal ordering is given by the following rule. Item (j) precedes item (j+1) if

$$\text{(I)} \qquad \max (K_j \; , \; K_{j+1}) < \max (K'_j \; , \; K'_{j+1}) \; .$$

If there is equality, either ordering is optimal, provided it is consistent with all the definite preferences (see case 4 in Lemma 2).

By subtracting $\sum_{1}^{j+1} A_i - \sum_{1}^{j-1} B_i$ from each term in (I), it becomes

$$\max (-B_j \; , \; -A_{j+1}) < \max (-B_{j+1} \; , \; -A_j)$$

or

$$\text{(II)} \qquad \min (A_j \; , \; B_{j+1}) < \min (A_{j+1} \; , \; B_j) \; .$$

This ordering is transitive (proof follows), thus leading to a sequence $S^*$, unique except for some indifferent elements.

Then $F(S^*) \le F(S_o)$ for any sequence $S_o$, since $S^*$ can be obtained from $S_o$ by successive interchanges of consecutive items, according to (II), and each interchange will give a value of F smaller than or the same as before.

**LEMMA 2:** Relation (II) is transitive.

Suppose $\min(A_1, B_2) \leq \min(A_2, B_1)$ and $\min(A_2, B_3) \leq \min(A_3, B_2)$. Then $\min(A_1, B_3) \leq \min(A_3, B_1)$ except possibly when item 2 is indifferent to both 1 and 3.

**PROOF:**

Case 1. $A_1 \leq B_2, A_2, B_1$ and $A_2 \leq B_3, A_3, B_2$.
Then $A_1 \leq A_2 \leq A_3$ and $A_1 \leq B_1$ so that $A_1 \leq \min(A_3, B_1)$.

Case 2. $B_2 \leq A_1, A_2, B_1$ and $B_3 \leq A_2, A_3, B_2$.
Then $B_3 \leq B_2 \leq B_1$ and $B_3 \leq A_3$ so that $B_3 \leq \min(A_3, B_1)$.

Case 3. $A_1 \leq B_2, A_2, B_1$ and $B_3 \leq A_2, A_3, B_2$.
Then $A_1 \leq B_1$ and $B_3 \leq A_3$ so that $\min(A_1, B_3) \leq \min(A_3, B_1)$.

Case 4. $B_2 \leq A_1, A_2, B_1$ and $A_2 \leq B_3, A_3, B_2$.
Then $A_2 = B_2$, and we have item 2 indifferent to item 1 and item 3. In this case 1 may or may not precede 3, but there is no contradiction to transitivity as long as we order item 1 and item 3 first and then put item 2 anywhere.

Using (II), there is an extremely simple, practical way of ordering the items in n steps.

**WORKING RULE:**

1. List the A's and B's in two vertical columns.

| i | $A_i$ | $B_i$ |
|---|---|---|
| 1 | $A_1$ | $B_1$ |
| 2 | $A_2$ | $B_2$ |
| . | . | . |
| . | . | . |
| n | $A_n$ | $B_n$ |

2. Scan all the time periods for the shortest one.
3. If it is for the first machine (i.e., an $A_i$), place the corresponding item first.
4. If it is for the second machine (i.e., a $B_i$), place the corresponding item last.
5. Cross off both times for that item.
6. Repeat the steps on the reduced set of 2n - 2 time intervals, etc. Thus we work from both ends toward the middle.
7. In case of ties, for the sake of definiteness order the item with the smallest subscript first. In case of a tie between $A_i$ and $B_i$, order the item according to the A.

To illustrate the method, the following somewhat extreme example is worked out. Consider

| i | $A_i$ | $B_i$ |
|---|---|---|
| 1 | 4 | 5 |
| 2 | 4 | 1 |
| 3 | 30 | 4 |
| 4 | 6 | 30 |
| 5 | 2 | 3 |

The rule gives an optimal sequence (5, 1, 4, 3, 2). The total delay time for this sequence is 4 units, and the total elapsed time is 47 units. If one reversed the order of the items, the total time would be 78 units, the worst value possible.

## THREE-STAGE PRODUCTION SCHEDULE

For three different machines or stages (at most one item at a time on each machine), the problem loses some of the nice structure of the two-stage case. The problem is formulated, however, and for the special cases where min $A_i \geq \max B_j$ or min $C_i \geq \max B_j$ the complete solution is found analogously to the two-stage problem.

LEMMA 3: An optimal ordering can be reached if we assume the same ordering of the n items for each machine.

By Lemma 2 the orders on the first and third machines can be made the same as that of the second, i.e., the first two machines have the same orders and the last two machines have the same orders. Thus the lemma is proved.

For four or more stages, the optimal scheduling may call for a shift in ordering of the items. Consider two items going through four stages with times listed below:

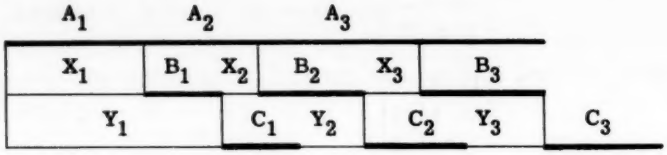
| i | $A_i$ | $B_i$ | $C_i$ | $D_i$ |
|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 3 |
| 2 | 3 | 1 | 1 | 3 |

It can be verified that the optimal scheduling here calls for a shift of ordering from the second to the third stage. Thus the general solution is apt to be very complicated.

NOTATION:

Let $A_i$ , $B_i$ , $X_i$ be defined as in the two-stage problem.
Let $C_i$ = setup time plus work time for the i-th item on the third machine.
Let $Y_i$ = the delay interval on the third machine immediately preceding the entry of the i-th item onto the third machine.

Consider the time scales for each machine.




We have

$$Y_1 = X_1 + B_1 = A_1 + B_1$$

$$Y_n = \max \left(\sum^{n} B_i + \sum^{n} X_i - \sum^{n-1} C_i - \sum^{n-1} Y_i , 0\right)$$

so that

$$\sum^{n} Y_i = \max \left(\sum^{n} B_i - \sum^{n-1} C_i + \sum^{n} X_i , \sum^{n-1} Y_i\right)$$

$$= \max \left(\sum^{n} B_i - \sum^{n-1} C_i + \sum^{n} X_i ,\right.$$

$$\left. \sum^{n-1} B_i - \sum^{n-2} C_i + \sum^{n-1} X_i , \ldots , B_1 + X_1 \right) .$$

Let

$$H_v = \sum^{v} B_i - \sum^{v-1} C_i , \quad v = 1, 2, \ldots , n,$$

and

$$K_u = \sum^{u} A_i - \sum^{u-1} B_i , \quad u = 1, 2, \ldots , n, \text{ as before.}$$

Then

$$\sum^{n} Y_i = \max_{1 \le u \le v \le n} (H_v + \max K_u) = \max_{1 \le u \le v \le n} (H_v + K_u) .$$

As before, we interchange the j-th and j+1-st items. Then the H's and K's are unchanged except possibly those with subscripts j and j+1.

Now we compare

$$\max (H_{j+1} + K_u , 1 \le u \le j + 1 ; H_j + K_u , 1 \le u \le j)$$

with

$$\max (H'_{j+1} + K'_u , 1 \le u \le j + 1 ; H'_j + K'_u , 1 \le u \le j) .$$

Notice these terms no longer involve just the subscripts j and j+1, and thus the decision is not independent of what precedes the interchanged elements.

## SPECIAL CASE WHERE MIN $A_i \geq$ MAX $B_j$

Here $\max_{u \leq v} K_u = K_v$, so that we now compare fewer terms. Our rule now states that the j-th item precedes the j+1-st item if

(III) $$\max (H_{j+1} + K_{j+1} , H_j + K_j) < \max (H'_{j+1} + K'_{j+1} , H'_j + K'_j) .$$

In case of equality, we make the ordering of indifferent items consistent with the ordering given by the definite inequalities.

Then by subtracting

$$\sum_{i}^{j+1} A_i - \sum_{i}^{j-1} B_i + \sum_{i}^{j+1} B_i - \sum_{i}^{j-1} C_i$$

from both sides of (III), it becomes

$$\max (-B_j - C_j , -B_{j+1} - A_{j+1}) < \max (-B_{j+1} - C_{j+1} , -B_j - A_j)$$

or

(IV) $$\min (A_j + B_j , C_{j+1} + B_{j+1}) < \min (A_{j+1} + B_{j+1} , C_j + B_j) .$$

LEMMA 4: Relation (IV) is transitive.

Proof is the same as for Lemma 2.

By the same arguments as before, we can reach an optimal sequence by successive interchanges of adjacent elements in any sequence following this rule. Thus we have

THEOREM 2: If min $A_i \geq$ max $B_i$, $1 \leq i \leq n$, then an optimal three-stage production schedule is given by the following rule: Item i precedes item j if

$$\min (A_i + B_i , C_j + B_j) < \min (A_j + B_j , C_i + B_i) .$$

If equality holds, the two items are indifferent and either is permissible, provided we order these items in a manner consistent with the orders given by the definite inequalities.

As in the two-stage case, there is a short working rule providing the optimal scheduling very quickly. Here $A_i$ is replaced by $A_i + B_i$, and $B_j$ is replaced by $B_j + C_j$.

Note that the same results hold if $\min C_i \geq \max B_j$ .

Another special three-stage case is when the two-stage rule applied to the first two stages gives the same ordering as that for the last two stages. Then this ordering is the optimal for the three-stage case.

An equivalent statement of the three-stage problem is as follows:

Notice that

$$\max_{u \leq v \leq n} (K_u + H_v + \sum_{i=1}^{n} C_i)$$

is the maximum sum of elements passed through on all "walks" in the time matrix from the upper left-hand corner to the lower right-hand corner, taking steps to the right or downward. The problem is to find a scheduling of items which minimizes this maximum walk.

This interpretation is useful in numerical work for three-stage problems but does not carry over to four or more stages.

As previously noted, the optimal ordering is not always the same on each stage when there are more than three stages. As a practical working rule, however, one could assume the same order for each stage and then use the "maximum walk" interpretation to eliminate candidates for an optimal schedule.

## REFERENCES

[1] Melvin E. Salveson, "On a Qualitative Method in Production Planning and Scheduling," Econometrica, Vol. 20, No. 9, (October 1952), pp. 571-574

[2] G. B. Dantzig, "Linear Model of Production," RAND Publication P-375A, February 1953.

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## NEWS AND MEMORANDA

RADM Victor D. Long, USN, Commander, Destroyer Flotilla One, Pacific, was assigned on September 2, 1953, to the Office of Naval Operations to be Director of the Logistics Plans Division. He relieved RADM Elmer E. Yeomans, USN, who became Commander, Cruiser Division Four, U. S. Atlantic Fleet.

A conference on the Theory of n-Person Games, arranged by the Princeton University Logistics Project, was held at Princeton University on March 20-21, 1953, under the direction of Professor A. W. Tucker. It consisted of nine talks, plus an afternoon discussion period. Copies of the minutes of the conference can be obtained from Professor A. W. Tucker, Fine Hall, Box 708, Princeton University, Princeton, New Jersey.

The Logistics Computer, a high-speed electronic digital computer constructed by Engineering Research Associates, Inc., for the Office of Naval Research, was installed at The George Washington University during March 1953. This Computer is being used by the Logistics Research Project of The George Washington University on research problems in logistics involving large quantities of data. Organizations of the Department of Defense desiring more information about the Logistics Computer should contact the Logistics Branch of the Office of Naval Research. Mass data-handling problems will be accepted for trial runs on the Computer to the extent which programming and machine time permits.

On August 24, 1953, Dr. F. Joachim Weyl was appointed Director of the Mathematical Sciences Division of the Office of Naval Research. As Director of this Division, Dr. Weyl will be in charge of four branches, one of which is the Logistics Branch. He succeeds Dr. Mina Rees who left to become the Dean of Faculty at Hunter College.

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The Fourth Annual Logistics Conference, sponsored jointly by the Office of Naval Research and the Logistics Research Project of The George Washington University, was held in Washington, D. C., on March 16-19, 1953. This conference has been called annually for the purpose of providing an exchange of information between the scientific groups of the country which are interested in naval logistics and the naval officers who are charged with the planning and operation of naval logistics.

The conference was divided into two parts, a "practical" and a "theoretical" session of two days each. The topics discussed in the practical session, under the chairmanship of

RADM Henry E. Eccles, USN (Retired), were Planning Problems and Techniques, and Logistics Data Processing. In the theoretical session, under the chairmanship of Professor Oskar Morgenstern of Princeton University, the topics were Distribution Control and Theory of Games. Over three hundred representatives of the armed services, other government agencies, universities, and private research organizations participated in the conference.

The general chairman, Dr. Mina Rees, opened the conference and introduced RADM C. M. Bolster, USN, Chief of Naval Research, and Dean Martin A. Mason of The George Washington University, who presented welcoming remarks for their organizations. Following these opening talks, the presentations given at the conference were as follows:

PLANNING PROBLEMS AND TECHNIQUES

1. Fleet Logistics
   Speaker: Vice Admiral F. C. Denebrink, USN
   Discussant: Rear Admiral J. E. Maher, USN
2. Supply in the Pacific Theater
   Speaker: Captain O. P. Lattu, SC, USN
   Discussant: Captain J. D. Parks, SC, USN
3. Maintenance and Repair in the Fleet
   Speaker: Rear Admiral W. D. Leggett, Jr., USN
   Discussant: Rear Admiral W. M. Hague, USN

LOGISTICS DATA PROCESSING

1. Survey of Modern Methods of Data Processing
   Speaker: Dr. E. W. Cannon
2. Data Storage Devices and Techniques
   Speaker: Jacob Rabinow
3. Digital Input and Output Devices for Automatic Computers
   Speaker: Dr. Nelson M. Blachman
4. A Description of the Logistics Computer
   Speaker: Lt. R. J. Rossheim, USNR
5. Applications of UNIVAC to Air Force Programming Problems
   Speaker: Emil Schell
6. Application of CRC Computer to Bureau of Aeronautics Problems
   Speaker: D. O. Larson
7. Application of the Logistics Computer to Naval Logistics Problems
   Speaker: J. Jay Wolf

DISTRIBUTION CONTROL

1. Optimal Technology for Supply Management
   Speaker: Rear Admiral Frederick L. Hetter, SC, USN
   Discussant: P. F. Hilbert
2. The Limits of Centralization
   Speaker: Dr. T. M. Whitin
   Discussant: Dr. M. E. Salveson
3. Estimating Shipping Requirements at Short Range
   Speaker: Dr. Harry M. Hughes
4. Optimal Scheduling in Transportation
   Speaker: Dr. I. Heller
   Discussant: Martin Shubik

THEORY OF GAMES

1. The Solution of Games by Behavior Strategies
   Speaker: Dr. H. W. Kuhn
   Discussant: Gerald Thompson
2. Reduction of Games in Extensive Form
   Speaker: Dr. Norman Dalkey
   Discussant: Lloyd Shapley
3. Blotto-Type Games
   Speaker: Dr. D. W. Blackett
   Discussant: Dr. W. H. Marlow
4. Machine Representation of a Symmetric Air War Game
   Speaker: Brigadier General L. I. Davis, USAF

The conference also served as the medium for the introduction and demonstration of the new Logistics Computer which was recently completed by Engineering Research Associates, Inc., for the Office of Naval Research.

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## RECENT PUBLICATIONS

SOVIET ECONOMIC GROWTH. Edited by Abram Bergson, Row Peterson & Co., 1953, 376 pp.

This is a report of the proceedings of an Arden House Conference of May 23-25, 1952, which discussed the factors in and prospects for long-run Soviet growth. The list of thirty-one participants and authors include eight consultants to or members of the RAND Corporation. The book consists of twelve presentations and discussions in the fields of National Income, Capital Formation and Allocation, Population and Labor Force, Transportation, Industrial Resources, Industrial Labor Productivity, Industrial Production, Agricultural Resources, Agricultural Organization, Agricultural Output and Employment, Soviet Economic Relations with the Orbit, and East-West Trade. The book contains several tables, footnotes, and an extensive index.

BEANS, BULLETS, AND BLACK OIL. By Rear Admiral Worrall R. Carter, USN (Retired), U. S. Government Printing Office, 1953, 482 pp.

This story of fleet logistics afloat in the Pacific during World War II was written by an individual who had extensive experience in this work. It contains 404 pages of text, an appendix of the names of commanding officers of the principal vessels engaged in logistic work, a special section of photographs of representative Service Force vessels, and a glossary of abbreviations. In addition, it contains forty-eight photographs of the Service Force in action, plus seventeen small area charts and a large fold-in chart of the Pacific Ocean. It is a comprehensive coverage of floating fleet support, which was one of the vital military developments in naval history.

STUDIES IN ECONOMETRIC METHOD. By Cowles Commission Research Staff Members, Edited by Wm. C. Hood and Tjalling C. Koopmans, John Wiley & Sons, Inc., 1953, 323 pp.

This book is No. 14 in a series of monographs by the Cowles Commission for Research in Economics. Some of the problems treated rather technically in the Cowles Commission Monograph No. 10, "Statistical Inference in Dynamic Economic Models," have been treated in an expository manner in this Monograph. The book, in addition, contains several new studies resulting from continuing research in econometric methods. The studies primarily report the results of research activity on econometric model construction, identifiability of parameters, estimation procedures, and the computation of estimates. The ten chapters in the book were written by Jacob Marschak, Tjalling C. Koopmans, Herbert A. Simon, Trygve Haavelmo, M. A.

Girshick and Trygve Haavelmo, Tjalling C. Koopmans and Wm. C. Hood, Herman Chernoff and Herman Rubin, S. G. Allen, Jr., Jean Bronfenbrenner, Herman Chernoff, and Nathan Divinsky.

THE CONTROL OF RAW MATERIALS. By J. Hurstfield, Her Majesty's Stationery Office and Longmans, Green, and Co., London, 1953, 530 pp.

This book is one in a series of British war production histories which is in turn part of a larger program of war histories. The book deals with the wartime problems relating to the flows of raw materials. The treatment is in four parts covering (a) the interwar period of preparation, (b) the wartime requirements for raw materials, (c) the problems of supply, including international coordination, and (d) the organization of control. Within the scope of 530 pages the author has not attempted a complete coverage of all problems of all raw materials. He has, however, presented a selection of fairly detailed histories of some of the particular problems which were encountered. The author attempts to seek general principles in the field of material control and also examines the more practical problems encountered in the actual administration and coordination of the machinery of material control. The supporting data are for the most part in the appendices which occupy some 80 pages.

CONTRIBUTIONS TO THE THEORY OF GAMES, Volume II (Annals of Mathematics Study No. 28, Princeton). Edited by H. W. Kuhn and A. W. Tucker, Princeton University Press, 1953, 395 pp.

This Study is a sequel to "Contributions to the Theory of Games," Volume I (Annals of Mathematics Study No. 24, Princeton 1950). Study No. 24 was divided into two parts, dealing with finite and infinite games, while this Study has been expanded into four parts: Part I, Finite Zero-Sum Two-Person Games, consisting of five papers; Part II, Infinite Zero-Sum Two-Person Games, consisting of five papers; Part III, Games in Extensive Form, consisting of six papers; and Part IV, General n-Person Games, consisting of five papers. There is an introduction at the start of each part describing the papers presented therein, along with their interconnections, and at the same time indicating the relationship of the four parts to one another. Volume II offers solutions to some of the problems posed in the preface to Volume I, providing a continuity of research between the two Studies. The twenty-one papers presented were prepared by twenty-two of the leading mathematicians working on game theory. The bibliography at the end of this Study supplements the one in Volume I.

INPUT-OUTPUT RELATIONS, PROCEEDINGS OF A CONFERENCE ON INTER-INDUSTRIAL RELATIONS HELD AT DRIEBERGEN, HOLLAND. Edited by The Netherlands Economic Institute, H. E. Stenfert Kroese N. V. Leiden, 1953, 234 pp.

This book consists of the papers presented at the conference, together with some of the discussions which took place. It is divided into two sections, "Theory" and "Applications." In the theoretical part, papers by W. Leontief, R. M. Goodwin, and T. C. Koopmans contain mainly summaries and simplified expositions of results already known. The discussions which followed

related to the uses of economic models, the aggregation problem, the estimation of parameters, and the mathematical form of the equations. In addition, there is a paper by W. Duane Evans entitled, "Indexes of Labor Productivity as a Partial Measure of Technological Change." In the applied part are papers on input-output analysis in Norway, the United Kingdom, and the Netherlands by O. Aukrust, T. Barna, and G. F. Loeb, respectively, and a short note by E. Glaser on the work sponsored directly by the U. S. Government in this field. There is a paper by J. Sandee and D. B. J. Schouten which attempts to combine a macro-economic model with a detailed input-output system. Finally there is an analysis by R. Stone and J. E. G. Utting relating input-output analysis to national accounting.

**THEORY OF GAMES AND ECONOMIC BEHAVIOR.** By John von Neumann and Oskar Morgenstern, 3rd Edition, Princeton University Press, 1953, 641 pp.

The new edition of this work which appeared originally in 1944 differs from the second edition only in the elimination of misprints discovered since 1947. The new preface refers to the increasing literature on game theory and to some new developments. Among these the authors mention "non-cooperative" games (i.e., games where the formation of coalitions encounters difficulties or is not considered by the players), linear programming, and the assignment problem. Finally a reference is made to the development of utility theory as presented in the first edition and extended further in the Appendix to the second edition (1947).

**THE THEORY OF INVENTORY MANAGEMENT.** By Thomson M. Whitin, Princeton University Press, 1953, 245 pp.

This book is divided into three parts, dealing with the subject of inventories in business enterprises, in the economy, and in the National Military Establishment. The main body of analysis lies in the first part which develops fundamental inventory control principles. In the light of these principles, the economic theory of the firm business cycle models, models of general economic equilibrium, and military inventory control systems are discussed. Inventory control policies are recommended which may lead to substantial saving for the private firm and for the military establishment. The book considers both theoretical and practical aspects of inventory control, emphasizing the importance of interrelating these two facets of the problem. It is contended that economic theorists have usually oversimplified their analysis by not explicitly considering inventories, although they are an element of critical importance for much of their analysis of the firm and the economy. In the area of military inventory control, the author recommends rigorous examination of inter-item differences in lead time, carrying charges, importance, and reliability of demand estimates.

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